APPLIED STATICS

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BY

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PREFACE TO SECOND EDITION.

It is the intention in the following pages to provide an elementary text to be used in conjunction with the lectures on Statics as given in the Faculty of Applied Science and Engineering at Toronto University. Recognizing the fact that the student very often wastes a great deal of time in working out the steps that are omitted in most mathematical text-books, the writer has endeavored to give each step in full, sometimes at the expense of conciseness.

Chapters XIV., XV., XVI. and the Appendix have been added to the original edition. The reader will no doubt notice many repetitions in the problems presented in the first three named chapters. These problems originally appeared in serial form in "The Canadian Engineer," which, of course, necessitated a large amount of repetition from week to week. It was thought better, however, to publish the problems in this book just as they were, for, as explained in the previous paragraph, this book is for the beginner and not the advanced student, who knows how and where to look for previous reasoning.

The writer wishes to acknowledge his indebtedness to Professor C. H. C. Wright, upon whose course of lectures the following work is based, and to Professor W. J. Loudon for many valuable suggestions.

THOS. R. LOUDON.

Toronto, August 5th, 1910.

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APPLIED STATICS

CHAPTER I.

MECHANICS.

In its general interpretation, the subject of Mechanics treats of the Laws of Motion, of the relations that exist between matter and motion, and of the laws of equilibrium of what are termed **forces.**

Force, although popularly regarded as a definite, concrete idea, is the same sort of thing as velocity or acceleration, and is measured by the rate at which a mass changes its velocity.

In Engineering Mechanics, however, it may be considered as a definite quantity, capable of being measured, and represented by a number, the unit of which is the weight of one pound avoirdupois.

We speak, therefore, in popular phraseology of a force of ten pounds, or of a force **acting on a body**, or of forces **acting in certain directions,** just as if a force were a concrete thing, these being merely convenient, although scientifically misleading, modes of expression, which have been so ingrained into the whole subject of Mechanics that it would require a vast amount of circumlocution to avoid their use.

The following subdivision of Mechanics is generally used:—

1. Kinematics, which treats only of the geometry of motion.

2. Dynamics, which treats of the relations between matter and motion.

3. Statics, which deals with the laws of equilibrium of forces.

Although the subject of Statics alone will be considered here, yet the student should bear in mind that the second section, that of Dynamics, treats of Newton's Laws of Motion, which are given here because of their importance, especially the Third Law, the applications of which are continually occurring in statical problems.

Newton's Laws of Motion.

I. A body continues in a state of rest or uniform motion in a straight line till disturbed from such a state by an impressed force.

II. Change of motion is proportional to the impressed force, and takes place in the direction of the impressed force.

III. **Action and reaction are equal and opposite.**

CHAPTER II.

STATICS.

Taking the definition of the subject of Statics as given in the last chapter, it is seen that the consideration of motion does not enter into statical problems. It may be said, however, that although the analysis of motion produced or the tendency to produce motion does not constitute a part of the subject of Statics, it is often very convenient to examine the motion, or tendency to produce motion, in order that the properties of a given force or set of forces may be intelligently obtained.

The only forces considered in this treatise are coplanar forces, or forces acting in one plane.

It is advisable, before attempting the discussion of statical problems, that the student familiarize himself with the meanings of the terms given below:—

Magnitude—A property which admits of being measured. Quantities having this property may be relatively compared.

Direction—The common idea of direction is in reality compounded of two ideas, viz.: The line along which a certain manifestation may take place, or along which one body lies relative to another body; and which **way** along the line the manifestation takes place, or the one body lies relative to the other body. Direction, in the following pages, will be understood to mean merely the line along which action is manifested; concisely, **line of action.**

Sense—One of two **ways** in which a magnitude may be described or generated along a direction.

This distinction between Direction and Sense is more clearly seen by examples:—

A body is said to move vertically upward; i.e., the body moves in the vertical direction with an upward sense.

A building is said to lie east of another building; i.e., the first building lies in the direction of a parallel of latitude relative to the other building, and in an easterly sense.

A Vector.—A line conceived to have fixed length and direction in space, but whose position is not limited: i.e., the line may be drawn anywhere.

A Vector Quantity.—Any quantity having the properties, Magnitude, Direction, and Sense, is known as a vector quantity; e.g., Force, Velocity, Displacement, Acceleration. Such quantities may be graphically represented by Vectors.

A Rigid Body is one whose original form cannot be altered; i.e., the body cannot be compressed, extended, broken, or bent. Such a body, of course, exists merely in theory.

CHAPTER III.

GRAPHICAL REPRESENTATIONS—THE RESULTANT—COMPONENTS.

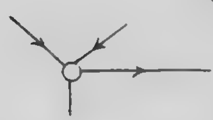
Graphical Representation of Force.

A Force, being a Vector Quantity, may be represented by a right line; for the direction of the line may represent the line of action of the force, the magnitude and sense being represented by first cutting off a portion of the line to some convenient scale of magnitude, and by then placing on this portion an arrow head pointed in the sense of action of the force.

Fig. 1.

Fig. 1 represents a force of ten pounds magnitude acting in the horizontal direction, with sense to the left, as indicated by the arrow head. The scale of magnitudes in this case is $\frac{1}{4}$ inch = 1 pound.

Fig. 2 represents a set of forces acting at a point. In this diagram the magnitudes of the forces are represented by the lengths of the various lines. It is often more convenient, however, merely to represent the directions and senses accurately, and to place the numerical values of the magnitudes beside the lines representing



Scale: 10pds. = 1in

Fig.2

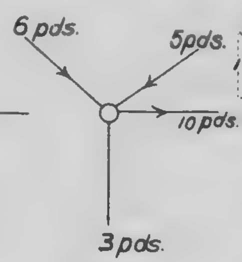


Fig.3

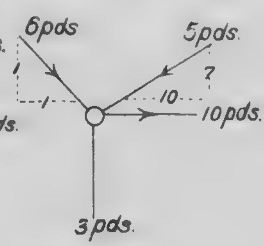


Fig 4

the respective forces as in Fig. 3. Sometimes, when general conditions are merely wished in the form of a note, the directions as well as the magnitudes are only relatively represented, and the actual direction ratios, as well as the magnitudes, jotted down on the diagram as in Fig. 4

A diagram, such as either Fig. 2, Fig. 3, or Fig. 4, in which is given all the relative information of a set of forces, will be referred to as the **Statical Diagram** of the set of forces in question.

Bow's Notation.

In order that the graphical representation of any particular force may be intelligently and concisely referred to, a system of lettering known as Bow's Notation will be used throughout the work whenever possible. In this notation, the areas between the lines representing the forces are lettered, and a force is known by the letters to the areas on each side of the line representing that particular force.

Referring to the Statical Diagram (Fig. 5), the force of 50 pounds is known as either the force AB or BA, the force of 30 pounds as either the force BC or CB, and the force of 60 pounds as either the force CA or AC

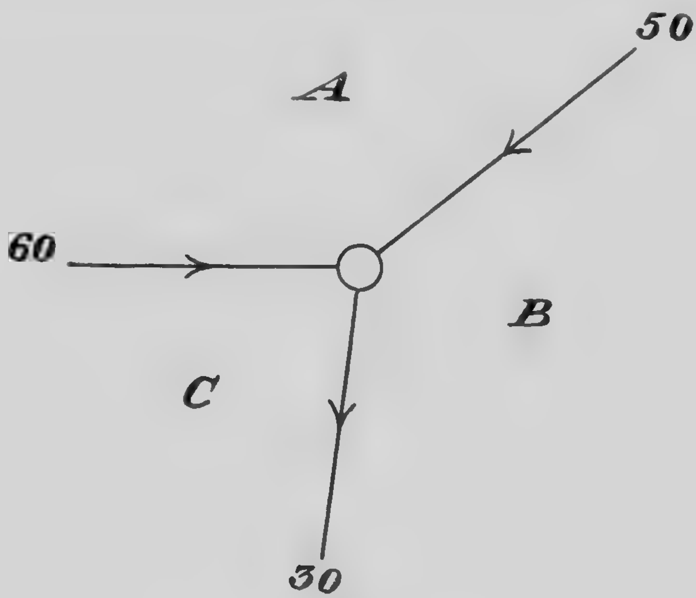


Fig. 5.

The Resultant Force is that one force which may replace and produce the same effect as a set of forces.

It is usual to refer to the Resultant Force merely as the **Resultant**.

It is evident that no resultant can be found for a set of forces in equilibrium. Because, if that could be done, there would be a single force (the resultant) acting on the body, and, therefore, motion would take place, which is impossible with equilibrium.

The Equilibriant or Balancing Force of a set of forces is that one force which, when acting with the set of forces, produces equilibrium.

The Equilibriant and Resultant of any given set of forces are of equal magnitude, and act in the same direction, but with opposite senses; or concisely, they are two equal and opposite forces. For, since the Resultant produces the same effect as the set of forces, it is manifest from definition that the Equilibriant would, when acting with the Resultant, produce equilibrium. Experimentally, it may be shown that the only possible manner in which two forces acting on the same body can give a state of equilibrium is that they be of equal magnitude and act in the same line with opposite senses.

Graphical Determination of the Magnitude, Direction, and Sense of the Resultant of a Set of Forces.

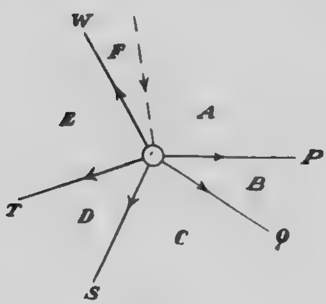


Fig. 6.

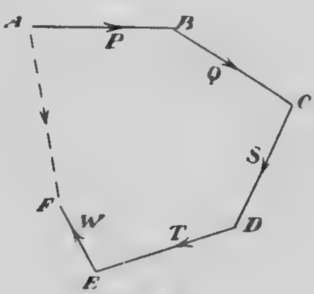


Fig. 6a.

Let there be given any set of forces, but, to make the problem clear, let us take a set of forces acting at a point. Fig. 6 is the Statical diagram of this set. Choose any initial point A as in Fig. 6a. From this point draw a line AB to represent **accurately** the force P in Fig. 6. From the point B, the line BC is drawn to represent the force Q (Fig. 6). In the same way the lines CD, DE, and EF are drawn to represent the forces S, T, and W, respectively. If now, the initial point A be joined to the final point F, then AF represents fully

the Resultant of the forces P, Q, S, T, and W. **The proof of this statement is experimental.**

The accuracy of the representation of the resultant in this last diagram depends entirely upon the care with which the relative directions and magnitudes of the forces are represented. If the directions of the forces relative to one another are represented accurately in the statical diagram, the simplest way of constructing the polygon is to draw the composing lines parallel to corresponding lines in the statical diagram.

The only restriction that is placed upon the construction of this polygon is that **the forces must be taken in such order that the sense marks in the polygon point continuously from the initial to the final point.** The sense mark of the resultant is always from the initial to the final point, and is, as it were, counter to the other sense marks.

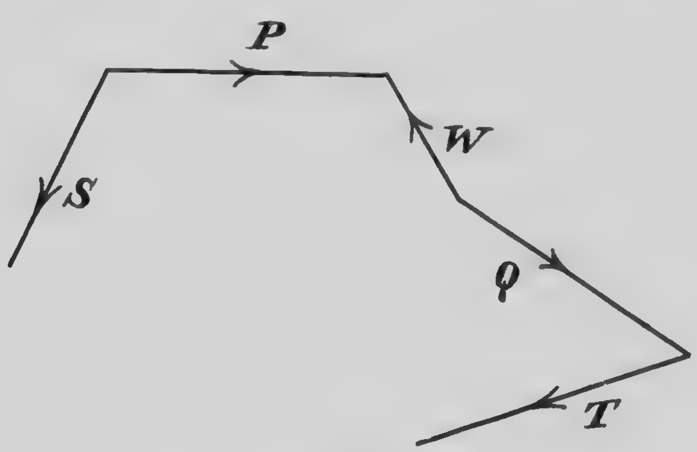


Fig. 7.

Fig. 7 shows an improper way of constructing the diagram. It will be noticed that the sense marks do not point continuously around the diagram from the initial to the final point. The resultant may, however, be found by a different arrangement of the representations than that shown in Fig. 6a. Fig. 8 shows a different diagram properly constructed with sense marks pointing continuously from the initial to final point. The resultant as found in this last case will have exactly the same magnitude, direction and sense as that found in Fig. 6a.

In the case where the forces do not act at a point, the procedure is exactly the same, the forces being chosen in such order that the sense marks on their representations point continuously from the initial to the final point of the diagram.

Diagrams such as Fig. 6a and Fig. 8 will be referred to in future as **Vector Polygons** or **Vector Diagrams,** and must not be confused with the Statical Diagram. The distinction lies in the fact that the Statical Diagram gives all information regarding the actual relative positions of the forces as they lie in space, as well as the properties of the forces; whereas, only the magnitudes, directions, and senses of the forces need be known to construct the Vector Polygon.

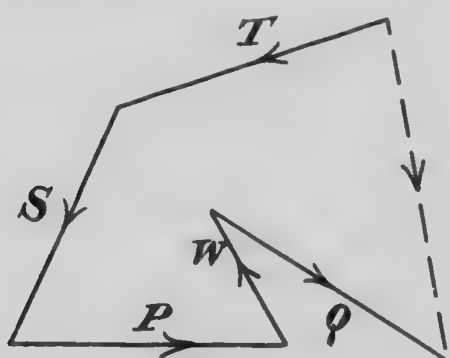


Fig. 8.

For a set of forces acting at a point, the resultant must act through this point in order to produce the same effect. Also, since the equilibriant acts in the same direction but in the opposite sense to the resultant, the equilibriant of a set of forces acting at a point must also act through the point. This gives a method of locating the resultant and equilibriant of a set of forces acting at a point, but for the general case, where the forces do not act at a point, the location of the resultant and equilibriant does not admit of such an easy solution, as will be seen later.

First Graphical Condition of Equilibrium.

It has been shown that a set of forces in equilibrium can have no resultant. Therefore, if the vector polygon be constructed for such a set of forces, the distance between the initial and final points, which represents the

resultant, must be equal to zero; i.e., the initial and final points will coincide, or, in other words, the polygon closes. From this, then, **the first graphical condition of equilibrium is that the vector polygon must close.**

Components.

It has been pointed out that a set of forces may be replaced by a single force. Conversely, a force may be replaced by a set of two or more forces. The forces comprising such a set are known as **Components** of the replaced force, and, since the force is interchangeable with any set of its components, it follows that **a force is the resultant of a set of its components.** This may be more clearly seen from Fig. 9.

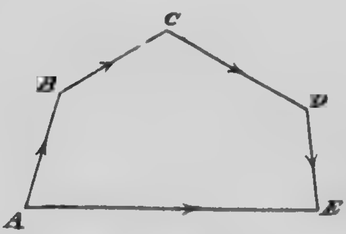


Fig. 9.

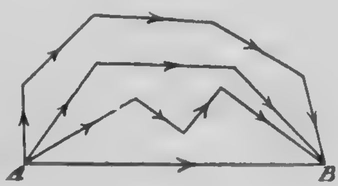


Fig. 10.

Let AE represent any force. AE is the resultant of AB, BC, CD, and DE; i.e., AE may replace AB, BC, CD, and DE. Conversely, the set AB, BC, CD, and DE may replace AE; i.e., they are a set of components. AE, the original force, is then the resultant of any of its component sets.

From this last statement it is seen that, if from the initial point of the representation of a force a number of lines be drawn continuously till the final point of the original representation reached, these lines will represent a set of components. Fig. 10 shows a number of sets of components to the force AB. It is easily seen that an infinite number of such sets may be found.

To any force there may be found an infinite number of sets of components.

Resolved Parts.

In many of the problems of Statics, it is convenient to consider a force as replaced by a set of components, and in order to give definiteness to the problem, it is

usual to replace the original force by two components whose directions are inclined at right angles to one another. Such a pair of components are known as **Resolved Parts,** and since a pair of resolved parts must always act perpendicularly to one another, the direction

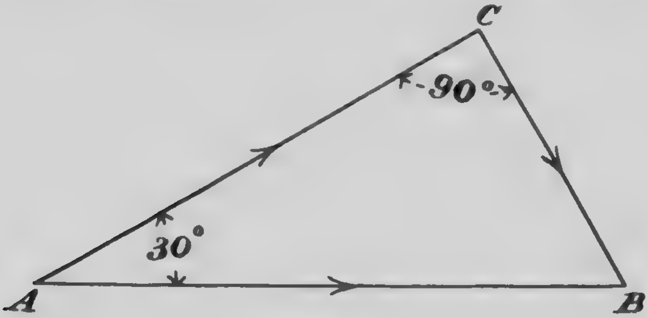


Fig. 11.

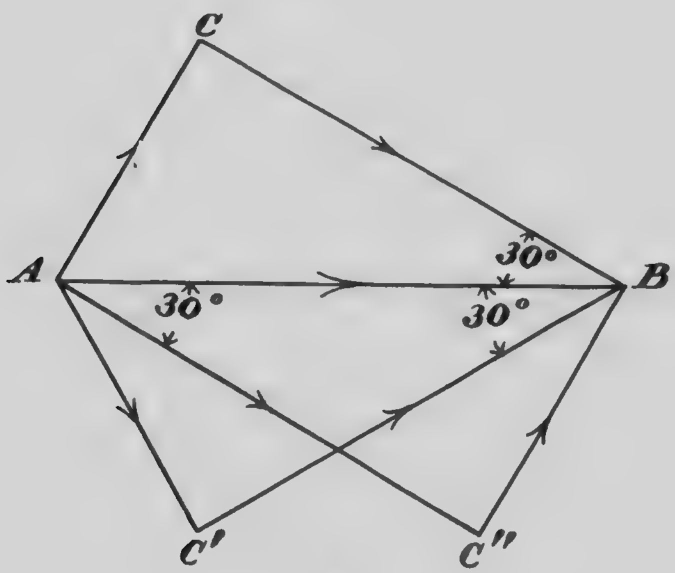


Fig. 12.

of one resolved part being given fixes the direction of the remaining resolved part. The operation of finding the value of a pair of resolved parts in any given directions that are perpendicular to one another, is known as **resolving** the original force in those directions.

Example:

A force of 50 pounds acts horizontally to the right. Find the magnitude of a pair of resolved parts, one of which is inclined to the horizontal at thirty degrees.

Until the student has become familiar with the operation of resolving forces, it is advisable to draw a rough graphical representation of the force and the required resolved parts, and then, from the data, find by trigonometrical means the required values.

Let AB (Fig. 11) represent the given force. AC and CB evidently represent the required resolved parts, for AC makes an angle of 30° with the horizontal, and CB is perpendicular to AC. Then

$$AC = AB \cos 30° = 50 \times \frac{\sqrt{3}}{2} = 25\sqrt{3}.$$

$$CB = AB \sin 30° = 50 \times \tfrac{1}{2} = 25,$$

which gives the magnitudes of the resolved parts in terms of the original force.

There are three other correct solutions to the problem which are shown in Fig. 12.

AC, CB; AC′, C′B; AC″, C″B are resolved parts which fill the requirements of the problem.

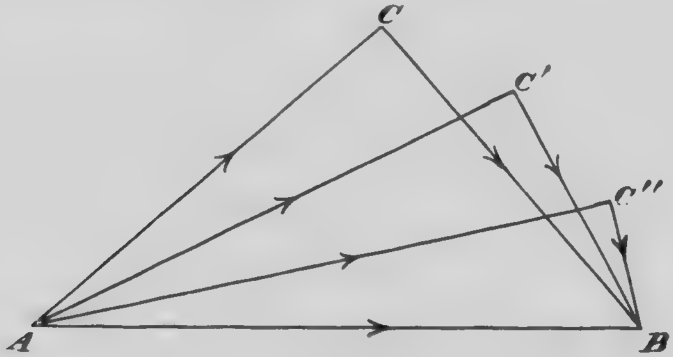


Fig. 13.

Resolved Parts Perpendicular and Parallel to a Given Force.

AC, CB; AC′, C′B, and AC″, C″B (Fig. 13) represent sets of resolved parts of AB. The directions of AC, AC′, and AC″ are respectively nearer to coincidence with the direction of AB. The directions of CB, C′B,

and C″B, on the other hand, are respectively more nearly perpendicular to AB. Also, AC″ is nearer the value of AB than AC, and C″B is closer to the value zero than CB. Consequently, if AB be resolved in directions coincident with and perpendicular to itself, the magnitudes of the resolved parts would be respectively equal to AB and zero. In general, then, **the magnitude of a resolved part perpendicular to a given force is equal to zero,** and **the corresponding resolved part, whose direction must be coincident with the given force, is equal to the given force.**

Horizontal and Vertical Resolved Parts.

In engineering statical problems, it is customary, unless otherwise stated, to confine attention to resolved parts in the horizontal and vertical directions, and in order that these may be concisely referred to, the horizontal resolved part is called the X, and the vertical resolved part the Y of the force in question. Furthermore, as will be seen, since the solution of many problems depends upon the algebraic summation of the X's and Y's of a set of forces, a convention as to the positive and negative sign of X's and Y's must be agreed upon.

Horizontal resolved parts (X's) acting to the right and vertical resolved parts (Y's) acting upward shall be considered positive.

Magnitudes of the X and Y of any Force whose Inclination to the Horizontal is Given.

Let AB (Fig. 14) represent a force of magnitude P. If the inclination of the line of action of the force to the horizontal be the angle α as shown, then AC and CB represent X_P and Y_P respectively.

$$X_P = AC = AB \cos \alpha = P \cos \alpha.$$
$$Y_P = CB = AB \sin \alpha = P \sin \alpha.$$

If P act to the left as represented by AB′, and the angle of inclination still remain α, X_P is negative, but its magnitude is still given by $P \cos \alpha$. The Y_P is positive and still of magnitude $P \sin \alpha$.

It is seen, then, that the magnitude of the X of any force, irrespective of its sign, is given by the **product of the magnitude of the force and the cosine of the**

angle of inclination to the horizontal. The magnitude of the Y of any force is given by the **product of the magnitude of the force and the sine of the angle of inclination to the horizontal.**

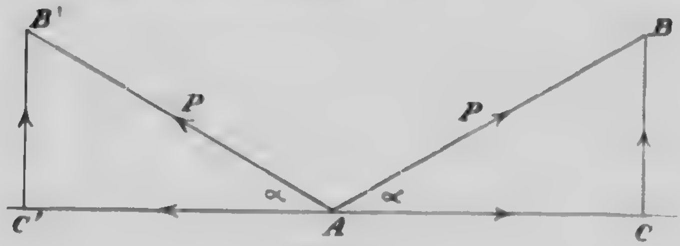


Fig. 14.

The X and Y of Horizontal and Vertical Forces.

If P be a horizontal force, then $\alpha = 0$, and

$$X_P = P\cos\alpha = P\cos 0^\circ = P.$$
$$Y_P = P\sin\alpha = P\sin 0^\circ = 0.$$

If P be a vertical force, then $\alpha = 90^\circ$, and

$$X_P = P\cos\alpha = P\cos 90^\circ = 0.$$
$$Y_P = P\sin\alpha = P\sin 90^\circ = P.$$

That is: **The horizontal resolved part of a horizontal force and the vertical resolved part of a vertical force are both equal to the force itself.**

The vertical resolved part of a horizontal force and the horizontal resolved part of a vertical force are both equal to zero.

CHAPTER IV.

MOMENTS—ANALYTICAL CONDITIONS OF EQUILIBRIUM—COUPLES.

If a body be pinned at some one point and acted on by any force not passing through this point, there will be produced a rotation, or a tendency to rotate. If a new point be chosen, or if a new force be introduced, this tendency to rotate will also change.

It may be shown that the turning effect varies as the product of the magnitude of the force and the perpendicular distance of the force from the point. To this product is given the name of the **Moment** of the force.

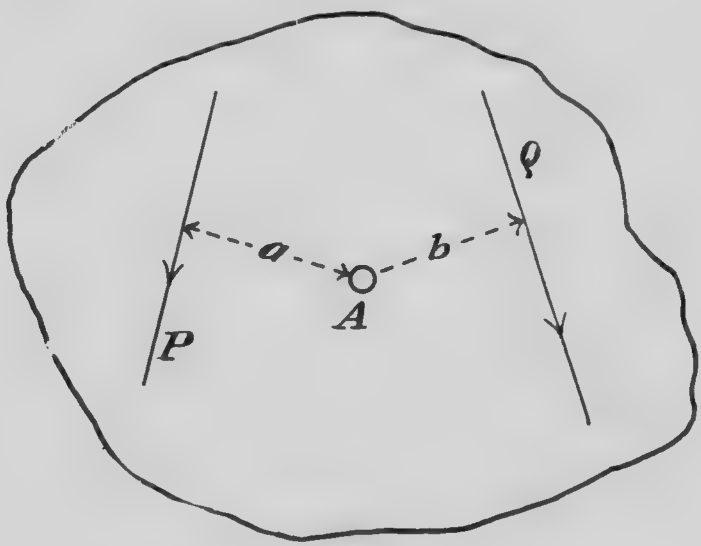


Fig. 15.

Definition:—

The moment of a force about a given point is the product of the magnitude of the force and the perpendicular distance of the point from the line of action of the force.

Since it is often necessary to find the algebraic sum of the moments of a set of forces about some one point, a convention as to the positive and negative sign of moments has to be agreed upon. In all the following problems, **forces which produce or tend to produce**

rotation of the body acted upon about the point in question in the sense of the motion of the hands of a clock have positive moments.

It may be said here that the other sense of rotation is very often used as the positive. It really does not matter which convention is used **as long as one convention is adhered to throughout the same problem.**

As an illustration, let P and Q (Fig. 15) represent two forces acting on a body, and let A be any point in the body, a and b being the perpendicular distances of P and Q, respectively, from the point A.

Imagining the body to be pinned at A, the force P would cause or tend to cause a rotation of the body in the counter clockwise sense of motion. The moment of P about A is, therefore, negative, and may be concisely written—

$$M_P = -P.a.$$

Under the same conditions, Q would cause or tend to cause a rotation of the body in the same sense as the motion of the hands of a clock. Q, in consequence, is said to have a positive moment, so that—

$$M_Q = +Q.b.$$

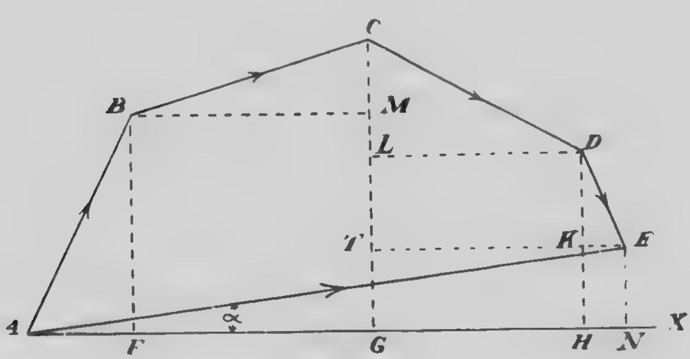


Fig. 16.

Analytical Determination of the Resultant.

To show that $\Sigma X = X_R$ and $\Sigma Y = Y_R$.

Let ABCDE (Fig. 16) be the Vector Polygon of a set of forces, the resultant of which it is required to find analytically.

Through A draw AX to represent the horizontal direction and from B, C, D, and E drop perpendiculars to AX, intersecting AX at F, G, H, and N, respectively. Through B, D, and E draw BM, DL, and KE parallel to AX, intersecting CG and DH at M, L, and K, respectively.

The horizontal and vertical resolved parts of the forces represented by AB, BC, CD, and DE are represented by the following lines:—

$X_{AB} = AF.$	$Y_{AB} = FB = GM.$
$X_{BC} = BM = FG.$	$Y_{BC} = MC.$
$X_{CD} = LD = GH.$	$Y_{CD} = -LC.$
$X_{DE} = KE = HN.$	$Y_{DE} = -KD = -TL.$

Denoting the algebraic sum of the horizontal resolved parts by ΣX and the algebraic sum of the vertical resolved parts by ΣY—

$$\begin{aligned}\Sigma X &= X_{AB} + X_{BC} + X_{CD} + X_{DE}.\\ &= AF + FG + GH + HN.\\ &= AN.\end{aligned}$$

But $AN = X_{AE}$ = horizontal resolved part of the resultant; i.e., $\Sigma X = X_R$.

$$\begin{aligned}\Sigma Y &= Y_{AB} + Y_{BC} + Y_{CD} + Y_{DE}.\\ &= GM + MC - LC - TL.\\ &= GT = NE.\end{aligned}$$

But $NE = Y_{AE}$ = vertical resolved part of the resultant; i.e., $\Sigma Y = Y_R$.

If the inclination of the line of action of the resultant AE to the horizontal be denoted by α, then—

$$\tan \alpha = \frac{NE}{AN} = \frac{\Sigma Y}{\Sigma X}$$

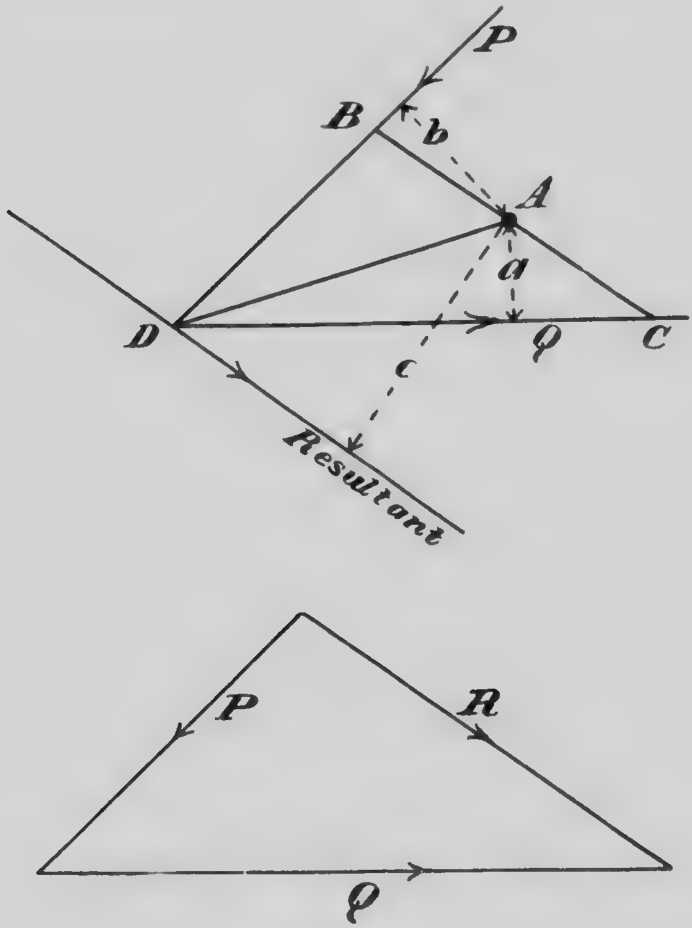
$$\text{and } R = AE = \sqrt{AN^2 + NE^2} = \sqrt{\Sigma X^2 + \Sigma Y^2}.$$

To show that $\Sigma M = M_R$.

Consider any two forces, P, Q, as in Fig. 17.

Determine, by means of the vector polygon, the resultant R of P and Q. This resultant will act at D, the intersection of the lines of action of P and Q.

Choose any point A about which to take moments, join A to D and through A draw a line parallel to the line of action of the resultant R, intersecting P and Q at B and C, respectively. The perpendicular distances of A from the lines of action of P, Q, and R are b, a, and c, respectively.



Vector Polygon

Fig. 17.

Because the lines BD, DC, and BC are situated relatively to one another, as are the directions of P, Q, and R, the triangle BDC may be taken as a vector

polygon of the forces P, Q, and their resultant R, so that—

$$P = BD;\ Q = DC;\ R = BC.$$

Taking moments about A

$$\begin{aligned} M_P &= -P.b = -BD.b = -2\ \triangle\ ABD. \\ M_Q &= -Q.a = -DC.a = -2\ \triangle\ ADC. \\ M_R &= -R.c = -BC.c = -2\ \triangle\ BDC. \end{aligned}$$

Denoting the algebraic sum of the moments of the forces P and Q by ΣM—

$$\begin{aligned} \Sigma M &= M_P + M_Q. \\ &= -2\ \triangle\ ABD - 2\ \triangle\ ADC. \\ &= -2\ \triangle\ BDC. \end{aligned}$$

But $-2\ \triangle\ BDC = M_R$.

Therefore, $\Sigma M = M_R$.

That is, the algebraic sum of the moments of two forces about any point is equal to the moment of their resultant about the same point.

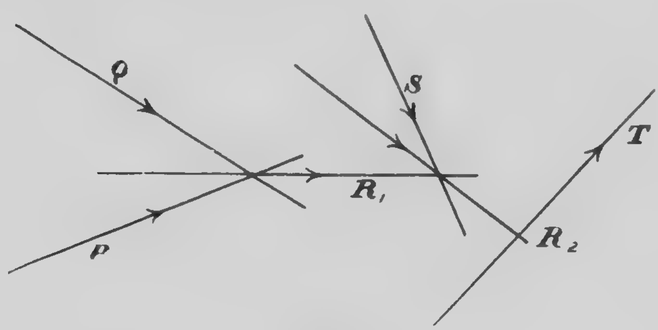


Fig. 18.

That the proof of the last proposition will apply to a case where the set of forces is more than two in number may be seen by considering a set of forces such as P, Q, S, and T represented in Fig. 18. P and Q may be replaced by their resultant R_1, so that R_1, S, and T are equivalent to the original set of forces. R_1 and S may be replaced by their resultant R_2, leaving R_2 and T as equivalent to the original four forces; that is, any number of forces comprising a set may be finally reduced to an equivalent pair of forces by such a system of combining and substituting resultants, and to the final pair

so found may be applied the preceding proof. **Therefore, $\Sigma M = M_R$ for any set of forces.**

In Fig. 17, the point A might have been taken in several other positions relative to P, Q, and R. If the point be taken in the line of action of either P or Q, the moment of either P or Q, respectively, would be zero. If the point be taken in the line of action of R, it is evident that a new construction must be devised. In this case, draw through the point a line parallel to either the force P or Q, producing it to intersect the line of action of the remaining force. The triangle so formed may be taken as a vector polygon as before and the same proof applied, the moment of R being zero.

Tabulating these analytical results:

$$\Sigma X = X_R. \qquad \tan \alpha = \frac{\Sigma Y}{\Sigma X} \qquad R = \sqrt{\Sigma X^2 + \Sigma Y^2}.$$
$$\Sigma Y = Y_R.$$
$$\Sigma M = M_R.$$

From these, the magnitude, direction, sense, and location of the resultant of any set of forces may be fully determined.

Analytical Conditions of Equilibrium.

It has previously been shown that a set of forces in equilibrium can have no resultant. From this it follows that since the value of the resultant is zero, the X, Y, and M of R must be equal to zero, and consequently:—

$$\Sigma X = 0.$$
$$\Sigma Y = 0.$$
$$\Sigma M = 0.$$

are the analytical conditions of equilibrium.

Couples.

Definition: A couple is a pair of equal forces which act in parallel directions with opposite senses.

Fig. 19 represents a couple. The two forces are of magnitude P, and are a distance a apart.

If the vector polygon be drawn for these two forces it must necessarily close.

Let α be the angle that the directions of the forces make with the horizontal.

Applying the analytical conditions ΣX, ΣY, and ΣM

$$\Sigma X = P \cos \alpha - P \cos \alpha = 0.$$
$$\Sigma Y = P \sin \alpha - P \sin \alpha = 0.$$

Taking moments about any point A distant x from one of the forces

$$\Sigma M = P\ (a + x) - P\ x = P.a.$$

If A be taken in any other position, it is found that the result is always the same, and since P and a are both constants—

$$\Sigma M = P.a = \text{Constant}.$$

Summary:

For a couple, then, the vector polygon must close.

$$\Sigma X = 0.$$
$$\Sigma Y = 0.$$
$$\Sigma M = P.a = \text{Constant}.$$

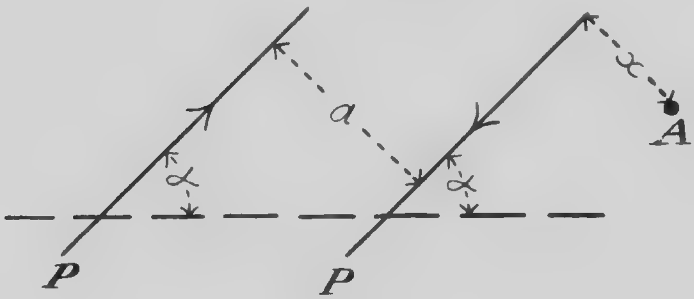


Fig. 19.

Three Forces in Equilibrium.

Three forces in equilibrium must have lines of action which intersect at a common point.

If it were possible, let P, Q, and S (Fig. 132) be three forces in equilibrium. Take moments about the point O, the intersection of the lines of action of P and Q

$$\begin{aligned} \Sigma M &= M_P + M_Q + M_S. \\ &= P.o + Q.o + S.a. \\ &= S.a; \end{aligned}$$

i.e., these forces cannot be in equilibrium, for ΣM must equal zero for a set of forces in equilibrium.

It is evident, however, that if the distance a becomes zero, that is, the line of action of S passes through O, that ΣM would also become equal to zero and the three forces would be in equilibrium, provided, of course, that the other conditions, $\Sigma X = o$ and $\Sigma Y = o$, were fulfilled.

(It may be mentioned here that three forces do not have to act at a point in order that ΣX and ΣY be equal

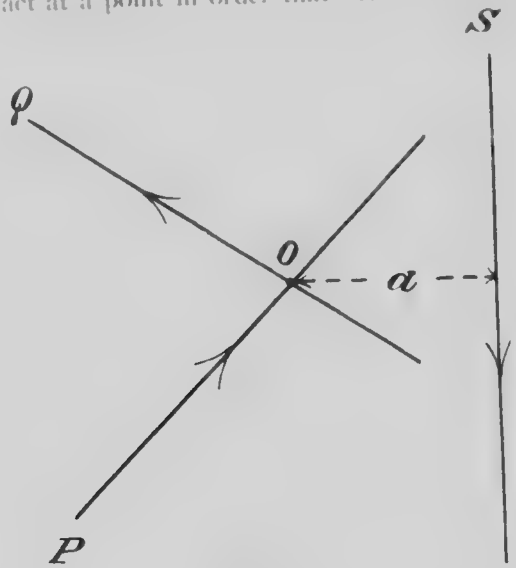


Fig. 132.

to zero. It is merely that ΣM may equal zero for three forces that the above condition must be fulfilled.)

The reader must clearly understand that every set of three forces which acts at a point is not in equilibrium; but if three forces are in equilibrium, they must fulfil the condition enunciated at the beginning of this proposition.

CHAPTER V.

DETERMINATION OF UNKNOWN FORCES TENSION AND COMPRESSION—STRESS IN TRUSS MEMBERS.

Let Fig. 20 represent a plank resting on an abutment A such that 10 feet and 30 feet of the total length overhang to the left and right, respectively. If 100 pounds be placed on the left hand end of the beam, what must be the magnitude of the force P exerted at the other end in order to preserve equilibrium, and what is the magnitude of the abutment reaction A?

Consider the beam as a rigid body. The forces acting on the body are: The abutment reaction, acting vertically upward with a magnitude of A pounds, and the two forces of magnitude, 100 and P pounds, acting

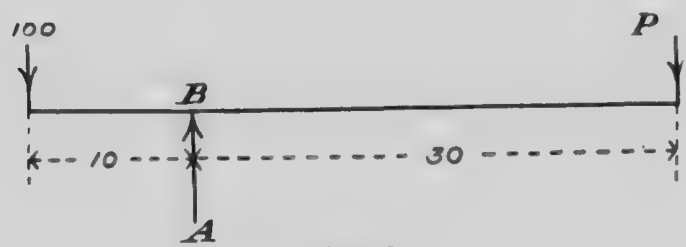


Fig. 20.

vertically downward, and these three forces are in equilibrium; therefore, $\Sigma X = 0$, $\Sigma Y = 0$, and $\Sigma M = 0$.

Apply these analytical conditions of equilibrium:—

$\Sigma X = X_{100} + X_A + X_P = 0.$

$0 + 0 + 0 = 0$ since all the forces act vertically.

$\Sigma Y = Y_{100} + Y_A + Y_P = 0.$

$-100 + A - P = 0.$

$A - P = 100 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$

Take moments about any point in the abutment, say, B. (The convention as to positive and negative moments must be kept in mind.)

$$\Sigma M = M_{100} + M_A + M_P = 0.$$
$$1000 + 10 \times 0 - P.30 = 0.$$
$$P = \frac{1000}{30} \quad \ldots \ldots (2)$$

Substituting this value of P found in (2) into (1),

$$A = \frac{1000}{3} - 100$$
$$A = 100 \quad \frac{100}{3}$$

It will be noticed that the equation $\Sigma X = 0$ gave absolutely no information, and also that if P had been the only quantity required, the equation $\Sigma M = 0$ would have given it; that is, it is not always necessary to use all three equations to find an unknown quantity. As to which one should be used for a given problem or whether two or all three have to be used, experience alone in working problems will give the student the ability to determine this at once by inspection. No mistake can be made, however, by using all three equations.

If a set of forces are in equilibrium, it does not signify about what point moments are taken; but it is best to choose a point in the line of action of one of the unknown forces, for by so doing the moment of that force becomes equal to zero, thus eliminating an unknown quantity and making it possible to solve the problem with one equation if there are only two unknown forces in question.

Abutment Reactions.

Fig. 21 represents a truss supported on two abutments; required to find the value of the abutment reactions AE and DC.

Let the unknown values of AE and DC be P and Q, respectively.

The truss is acted upon by a set of outside forces which are in equilibrium; therefore, for this set of forces, $\Sigma X = 0$, $\Sigma Y = 0$, and $\Sigma M = 0$.

Take moments about any point in the line of action of either abutment reaction, say, DC.

$$\Sigma M = M_{AE} + M_{AB} + M_{BC} + M_{CD} + M_{DF} = 0.$$
$$P.40 - 100.30 - 100.10 + Q.0 - 100.20 = 0.$$
$$P.40 - 100\,(30 + 10 + 20) = 0.$$
$$P = 150$$
$$\Sigma Y = Y_{AE} + Y_{AB} + Y_{BC} + Y_{CD} + Y_{DE} = 0.$$
$$150 - 100 - 100 + Q - 100 = 0.$$
$$Q = 150$$

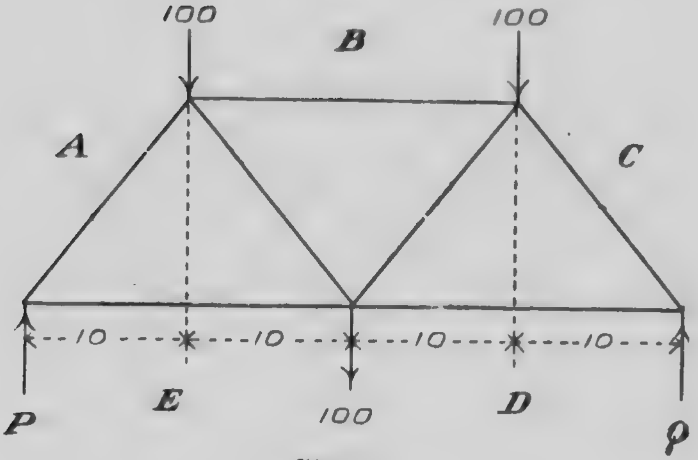


Fig. 21.

The use of the last equation is, in a sense, superfluous, for since the abutments support all the load, and if one abutment be found to support 150 pounds, then the other must support the remainder of the load. It is well for the student, however, to see that this reasoning is really based upon the condition $\Sigma Y = 0$.

Tension and Compression.

If a steel bar AB (Fig. 22) rest against a wall and a resultant force P be exerted at B against the bar, the bar is said to be **in compression.**

Keeping in mind Newton's third law of motion, that action and reaction are equal and opposite, it is evident that at B there must be a force exerted contrary and equal to P. This is indicated by D. If any section such as C be examined, it also follows that the right hand particles must be exerting a force equal to P, which is resisted by the left hand particles, as indicated by E and F. Arriving at the end A, there is exerted a push against the wall of magnitude P as represented by G.

If a steel bar AB (Fig. 23) be affixed to an abutment and a resultant force P' be exerted so as to tend to pull the bar away from the abutment, the bar is said to be **in tension.**

At B there must be exerted an equal and opposite force to P as indicated by D, and at the end A the bar exerts a pull on the abutment as shown by G. If any section C be considered, it follows that the material tends to tear apart; that is, the forces acting at the section must be such as represented by E and F.

In both these cases the bars are said to be stressed, stress being a resistance to change of shape, and must not be confused with the term strain, which refers to the change in dimension due to stress.

In general, **If at any section of a body it is found that the resultant of the forces acting at the section acts against the section, the body is said to be in compression; or, if this resultant is found to be acting away from the section, the body is in tension.**

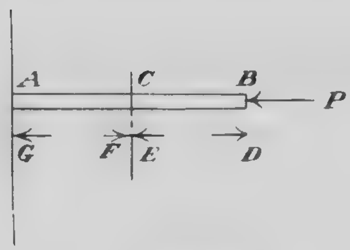


Fig. 22.

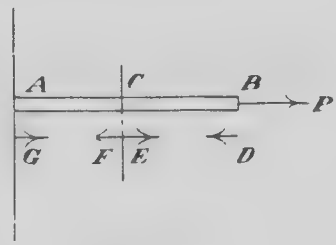


Fig. 23.

The Stress in Truss Members.

The following problem indicates the procedure in determining stresses in the members of a framed structure. The truss chosen to illustrate this is known as a cantilever, the principle of which is applied in bridge building the world over. The example given is necessarily a very simple case, but, while difficulties may arise in other problems relating to stress in truss members, they are found to be due to lack of knowledge as to where to begin the problem rather than to any change in the principle involved. Experience in working problems is the only remedy for this last-named obstacle.

Graphical Solution.

Fig. 24 represents a cantilever supporting a load of 100 pounds.

Consider the forces acting at the point ABC. In order to fix one's ideas on the point in question, it is always advisable to draw a statical diagram for that point, placing on it all the known data.

Fig. 25 is the statical diagram for the point ABC. The forces acting at this point are in equilibrium, one being completely known, the other two, AB and CA, being only partially known; i.e., their directions are known. **Such forces as AB and CA will always be referred to as unknown forces.**

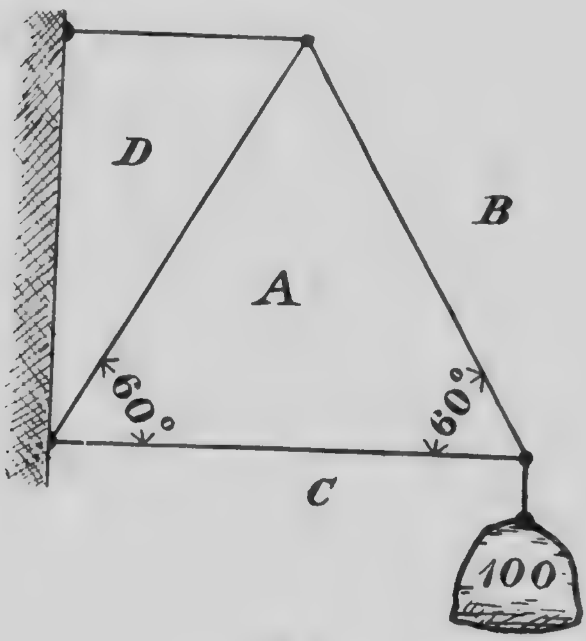


Fig. 24.

From any initial point B (Fig. 26) draw a line parallel to BC in Fig. 25 to represent the direction of the force BC. Cut off BC, in Fig. 26, to represent a magnitude of 100 pounds, placing the sense mark from B to C, thus totally representing the force BC.

From C (Fig. 26) draw a line parallel to CA in Fig. 25. Now, since the magnitude of the force CA is

unknown, it is at present seemingly impossible to locate the point A on this new line, so that CA may represent the force CA. However, it is known that the forces BC, CA, and AB are in equilibrium, from which it follows that if the vector polygon be drawn for these forces it must close. So, then, from some point in the last line drawn from C a third line must be drawn parallel to AB (Fig. 25), such that it will pass through B (Fig. 26), thereby closing the vector polygon. This is best accomplished by drawing through B a line parallel to AB (Fig. 25), intersecting the other line last drawn, at A. It is evident from the previous discussion

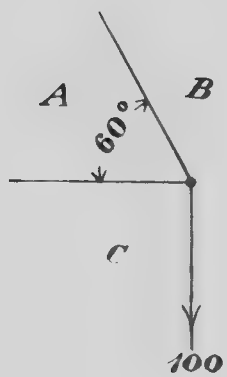


Fig. 25.

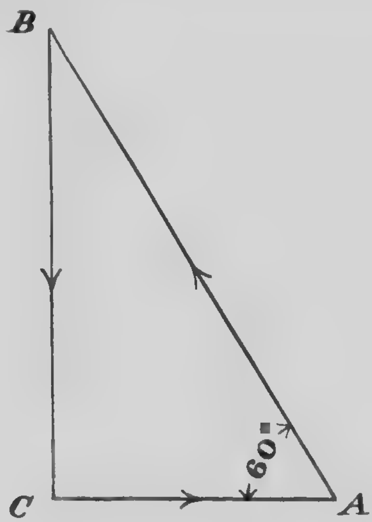


Fig. 26.

that CA and AB (Fig. 26) represent completely the hitherto unknown forces CA and AB, their senses being such as to give continuous sense from the initial point to the final point of the polygon as indicated.

From the data on the statical diagram the triangle BCA is a 60° right angled triangle. It follows that since BC represents 100 pounds, CA and AB represent $\frac{100}{\sqrt{3}}$ and $\frac{200}{\sqrt{3}}$ pounds, respectively.

Placing the sense of CA and AB as found in the vector polygon, on the statical diagram, it is seen that

CA acts against the point and AB away from the point. The member CA is, therefore, in compression and AB in tension to the extent of $\frac{100}{\sqrt{3}}$ pounds and $\frac{200}{\sqrt{3}}$ pounds, respectively. (For the reasoning on tension and compression see page 34.)

Fig. 27 is the statical diagram for the point ABD. Since the member AB is in tension, there is a known force AB acting away from the point ABD as indicated. The two forces BD and DA are unknown.

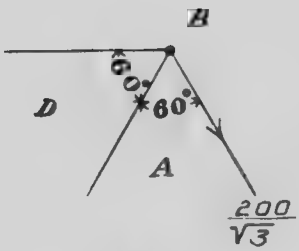
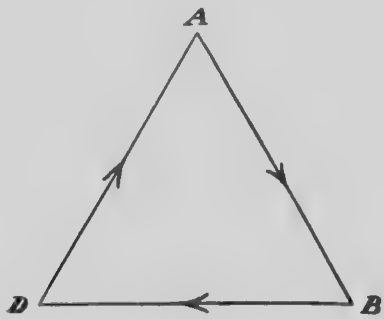


Fig. 27.



ig. 28.

Referring to Fig. 28, AB represents the known force AB. From B is drawn a line to represent the direction of the unknown force BD, and from A a line is drawn to represent the direction of the second unknown force DA. These two lines intersect at D. BD and DA represent the unknown forces BD and DA, and because the triangle ABD is equilateral (from data), BD and DA must each be equal to AB; i.e., of $\frac{200}{\sqrt{3}}$ pounds magnitude. BD acts away from the point ABD and DA against the point. The members BD and DA are, therefore, in tension and compression, respectively.

The results of the preceding solution are given in Fig. 29, compression and tension members being shown by thickened and light lines, respectively.

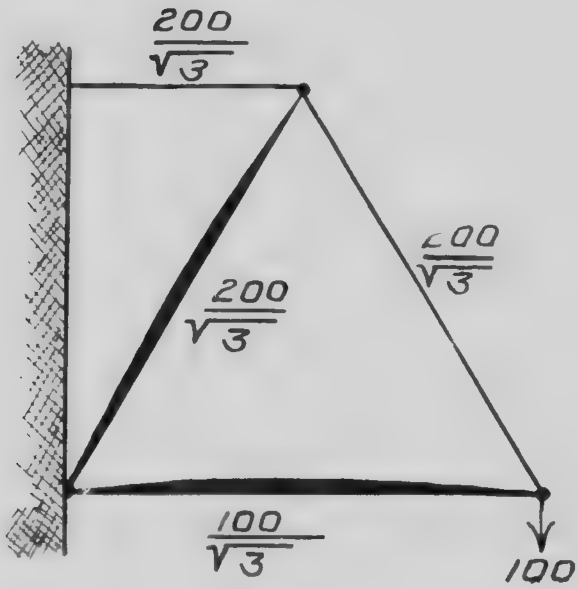


Fig. 29.

Analytical Solution.

Apply the analytical conditions of equilibrium to the points ABC and ABD.

Point ABC:—

$$\Sigma Y = Y_{CB} + Y_{BA} + Y_{AC} = 0.$$
$$-100 + BA \sin 60^\circ + 0 = 0.$$
$$BA = \frac{200}{\sqrt{3}}$$

This positive result shows that the Y_{BA} is positive. BA, therefore, acts away from the point ABC; i.e., the member BA is in tension $\frac{200}{\sqrt{3}}$ pounds.

$$\Sigma X = X_{CB} + X_{BA} + X_{AC} = 0.$$
$$0 - BA \cos 60^\circ + AC = 0.$$
$$-\frac{200}{\sqrt{3}} \cdot \frac{1}{2} + AC = 0.$$
$$AC = \frac{100}{\sqrt{3}}$$

From the positive result, the X_{AC} is positive; i.e., AC acts against the point. The member AC is, therefore, in compression $\frac{100}{\sqrt{3}}$ pounds.

Point ABD:—

$$\Sigma Y = Y_{AB} + Y_{BD} + Y_{DA} = 0.$$
$$-AB \sin 60^\circ + 0 + DA \sin 60^\circ = 0.$$
$$DA = AB$$
$$= \frac{200}{\sqrt{3}}$$

DA is in compression $\frac{200}{\sqrt{3}}$ pounds.

$$\Sigma X = X_{AB} + X_{BD} + X_{DA} = 0.$$
$$AB \cos 60^\circ + BD + DA \cos 60^\circ = 0.$$
$$BD = -(AB + DA) \cos 60^\circ.$$
$$= -\left(\frac{200}{\sqrt{3}} \cdot \frac{200}{\sqrt{3}}\right). \frac{1}{2}.$$
$$= -\frac{200}{\sqrt{3}}$$

BD is in tension $\frac{200}{\sqrt{3}}$ pounds.

In the last two equations the reasoning as to why the members are in compression and tension has been left out. The student is advised to work this out himself from the reasoning in the fromer equations for the point ABC.

These results, found analytically, check those found graphically. It will be noticed that in the analytical solution the use of the equation ΣM was not necessary. Cases will arise, however, where all three equations have to be used, and only experience can teach which equation should be first used to the greatest advantage in getting quick results. No mistake can be made in using any one of the equations first, but it will soon be seen that simultaneous equations may often be avoided.

CHAPTER VI.

THE EQUILIBRIUM OR FUNICULAR POLYGON.

The Vector Polygon gives the magnitude, direction, and sense of the Resultant of a set of forces, but does not determine the true position of the line of action of this Resultant. The problem of locating the Resultant may be accomplished by means of what is termed the Equilibrium Polygon.

Let P, Q, and S (Fig. 149) be any set of co-planar forces, the Resultant of which it is required to locate.

In a previous discussion, it was shown that the size and shape of the body acted upon can in no way alter the position of the line of action of the Resultant and Equilibriant of a set of forces; in other words, **the Resultant and Equilibriant are independent of the size and shape of the body acted upon.** This being the case, it will be shown that the position of the Equilibriant and Resultant may be determined by replacing the original body by an unbraced, free-jointed frame, the shape of which will give the desired information.

ABCD (Fig. 150) is a Vector Polygon for the forces P, Q and S. DA evidently represents the Balancing Force or Equilibriant of this set of forces.

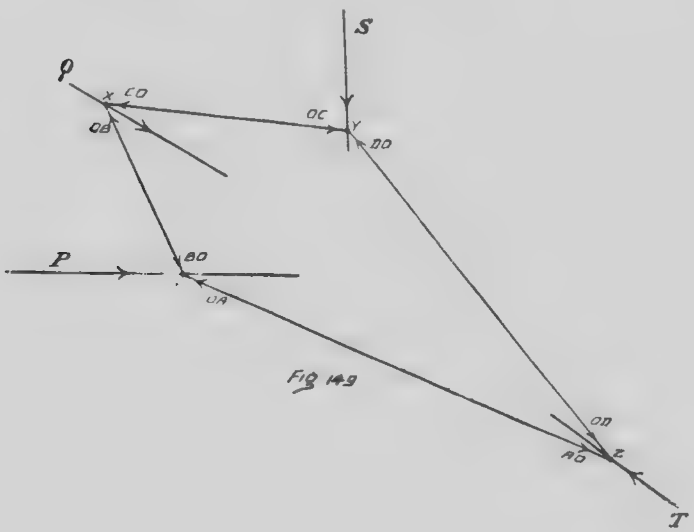


Fig 149

It is evident that **if the Equilibriant can be located, the position of the Resultant will also be known,** for the Equilibriant and Resultant of a set of forces have the same lines of action.

Choose any point O in Fig. 150 and join it to A, B, C, and D; i.e., to the terminations of the lines composing the Vector Polygon.

It is evident, from Fig. 150, that since the lines AB, BO, and OA form a closed polygon, the three forces represented by these lines will be in equilibrium, provided their lines of action intersect at a common point. If these three forces are in equilibrium, such a condition of affairs could be expressed by saying that any two of the forces balance the third. For instance, the forces BO and OA may balance the force AB; i.e, balance the force P.

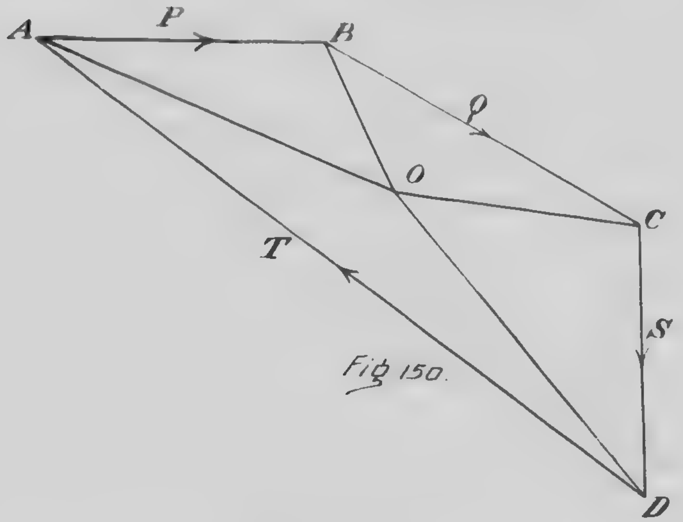


Fig 150.

At any point W on the line of action of P (Fig. 149) introduce the forces OA and BO. These three forces are in equilibrium.

For the time being, any discussion as to how these forces OA and BO are exerted and what constitutes the body acted upon by the three forces, P, BO, and OA, will be dropped.

Produce the line of action of BO to intersect the line of action of Q at X, and at this point introduce the forces represented by CO and OB (Fig. 150). The line of action of OB evidently coincides with BO (Fig. 149). It is seen

from a consideration of the triangle BCO (Fig. 150) that the forces OB and CO acting at X (Fig. 149) will balance the force Q.

Produce the line of action of CO to intersect the line of action of S at Y. At Y introduce the forces DO and OC represented by the lines DO and OC (Fig. 150). These forces balance the force S.

Produce the line of action of DO to intersect at Z the line of action of OA produced, and at Z introduce two forces, AO and OD, represented by AO and OD (Fig. 150).

Let the polygon WXYZ (Fig. 149) represent a frame having pin joints at W, X, Y, and Z. It is evident that the frame will be in equilibrium if some force be introduced at Z which will balance AO and OD. The force T, represented by DA (Fig. 150) evidently fulfils this condition.

It is now seen that a frame structure of the shape WXYZ would be in equilibrium under the action of the forces P, Q, S, and T; and the forces which were spoken of as being introduced at W, X, Y, and Z are really the forces exerted by the members on the joints of the frame. Furthermore, it is seen that the force T balances the forces P, Q, and S; i.e., the force T, which is the Equilibriant of P, Q, and S, has been located, which also locates the required Resultant.

The polygon WXYZ (Fig. 149) is known as an Equilibrium or Funicular Polygon.

The Balancing Couple.

In the preceding case the set of forces under discussion had a Resultant. Cases arise, however, where the Vector Polygon of a set of forces closes and yet the forces are by no means in equilibrium. It is evident that the forces P, Q, and S (Fig. 32) constitute such a case, for if their lines of action be produced, it is seen that they do not intersect at a point, and yet the Vector Polygon ABC closes.

Construct the Funicular Polygon for these forces, beginning by balancing S by CO and OA, and so on. It is seen that the lines of action of OA and AO do not intersect, but are parallel.

The reader must be careful at this point to distinguish between what has actually been accomplished from what has apparently been done. Apparently, the forces S, Q, and P have been balanced by their respective pairs of balancing forces CO, OA, and so forth. But in

order that these balancing forces be exerted, there must be some bodies acting. In the construction of the Equilibrium Polygon, it was seen that these forces were exerted by the members of a jointed frame. There must, then, be members with joints at D, E, and F (Fig. 32), and, in order that the forces OA and AO be exerted at D and F, respectively, there would necessarily be two members, GD and HF, parallel to one another.

It is readily seen now that if the members GD and HF are left free, the structure is by no means in equilibrium, and, therefore, the forces S, Q, and P are not

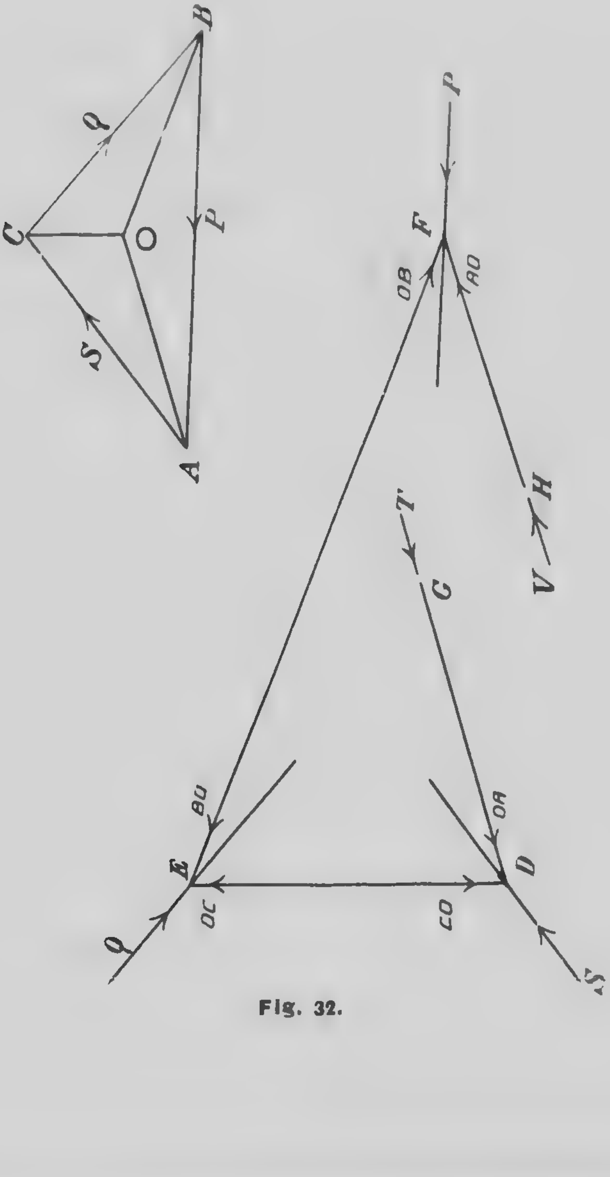


Fig. 32.

balanced. In order to bring about equilibrium there must be exerted at G a force equal to OA and at H a force equal to AO as indicated by T and V.

From the above discussion it is seen that T and V constitute a couple, since OA and AO are equal and opposite forces with parallel lines of action. To this couple is given the name of a **Balancing Couple.**

Resultant Couple.

The Resultant Couple to a set of forces would be equal and opposite to the Balancing Couple.

Second Graphical Condition of Equilibrium.

It is evident in the last proposition that if GD and HF were coincident that the frame, which would then be of the form DEF, would hold the forces S, Q, and P in equilibrium. The forces S, Q, and P in such a case would themselves necessarily be in equilibrium since no other external force was introduced into the system to bring about this equilibrium.

If, then, the lines of action of the first and last balancing forces have coincident directions, the original set of forces must be in equilibrium, which constitutes a second graphical condition of equilibrium. This is often more concisely stated by saying that **the Equilibrium or Funicular Polygon must close,** bearing in mind the meaning of the word "close" in this particular instance.

Graphical and Analytical Determination of the Resultant.

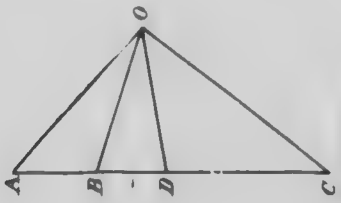
Graphical.	Analytical.
Vector Polygon gives the magnitude, direction, and sense of the Resultant.	$\Sigma X = X_R$. $\Sigma Y = Y_R$. $R = \sqrt{\Sigma X^2 + \Sigma Y^2}$. $\tan \alpha = \dfrac{\Sigma Y}{\Sigma X}$
Equilibrium Polygon locates the line of action of the Resultant.	$\Sigma M = M_R$.

Conditions of Equilibrium.

Graphical.	Analytical.
Vector Polygon must close.	$\Sigma X = 0$. $\Sigma Y = 0$.
Equilibrium Polygon must close.	$\Sigma M = 0$.

Properties of the Equilibrium Polygon.

Let P, Q, S, and T (Fig. 151) be a set of forces whose Vector Polygon is AB, BC, CD, and DA.



Vector Polygon

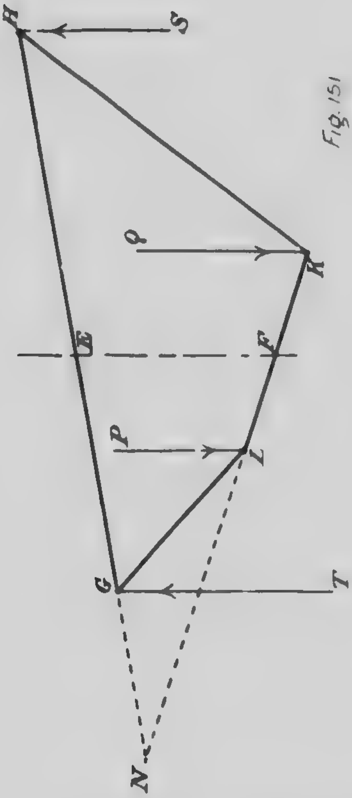


Fig. 151

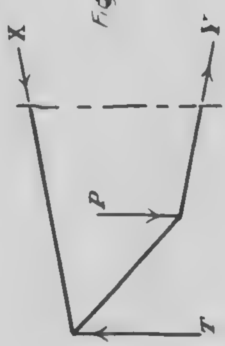


Fig. 151 a

P, Q, S, and T are held in equilibrium by the jointed frame, or Equilibrium Polygon GHKL.

Consider the portion of the frame to the left of any section EF, which in this case cuts the members GH and LK at E and F respectively.

The forces acting on the portion of the frame being considered are the external forces T and P and the forces exerted at E and F by the portions of the members GH and LK to the right of the section; and, since the member GH is in compression, there is exerted at E a force of magnitude OD acting against the section, and, because LK is in tension, there must be exerted at F a force of magnitude BO acting away from the section. This condition of affairs is shown at Fig. 151a, the forces acting at E and F being represented by the two forces X and Y respectively.

Now, it is evident that the forces X and Y (Fig. 151a) hold the portion of the frame to the left of the section in equilibrium against the action of the forces T and P. The resultant of X and Y, therefore, would accomplish the same result against the resultant of T and P, and, if these two sets of forces were replaced by their respective resultants, it is easily seen that these resultants would have the same lines of action if they counteract one another.

Produce GH and LK to intersect at N (Fig. 151).

Now, since the resultant of two forces always acts through the intersection of their lines of action, it is evident that the resultant of X and Y, and consequently the resultant of T and P, acts through the point N.

From the preceding discussion it is seen that if a section be taken through two members of the jointed frame, **the forces exerted on one side of the section by the cut members hold in equilibrium the external forces acting on the portion of the frame lying to the other side of the section.**

Also that, **the resultant of the external forces to one side of the section acts through the intersection of the cut members produced.**

CHAPTER VII.

THE METHOD OF SECTIONS.

It is often necessary to find the stress in one or more members of a truss, irrespective of the stress in other members. To obviate the tedious operation of working from point to point till the stress in the required member is finally reached, what is known as the **Method of Sections** is used, whereby the stress, say, in EF (Fig. 33) may be arrived at immediately **from a consideration of the forces acting on the truss.**

Imagine the truss represented in Fig. 33 as cut into

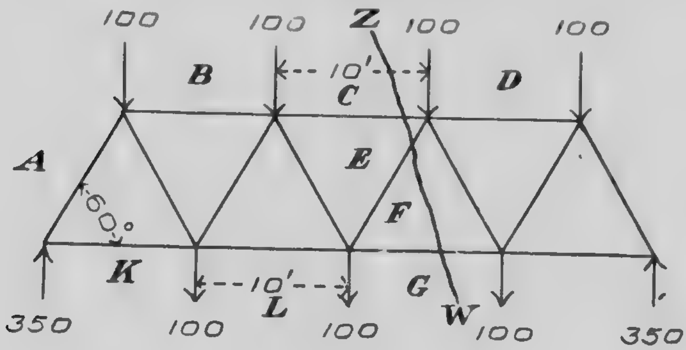


Fig. 33.

two sections at ZW. Consider the portion of the truss to the left of ZW. This considering the portion to the left is purely arbitrary. The same results would eventually be reached by considering the right-hand section, but in order to avoid confusion, the convention of considering the left-hand portion is agreed upon.

This portion of the truss to the left of ZW is held in equilibrium by certain forces **acting on it.** There is no difficulty in seeing that the left-hand abutment reaction AK, and the four loads AB, BC, KL, and LG, act on the body to left of ZW. **But that these forces alone could not preserve equilibrium** is easily seen. There must, then, be other forces acting on the body, and the only possible places where forces could act is where the portions of the members CE, EF, and FG, to the right of ZW, touch the portions to the left.

As far as the forces acting on the body are concerned, it does not matter whether the body be con-

sidered as a truss form or as a solid body having the boundary outline of the truss. It is generally less confusing, however, to choose the latter method, because it seems to concentrate attention on the extraneous forces.

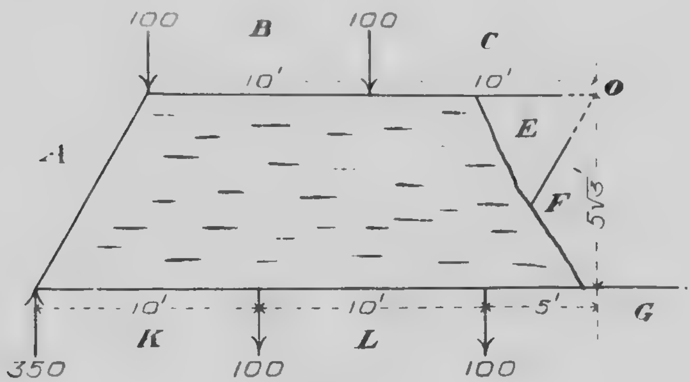


Fig. 34.

Fig. 34 represents the forces acting upon the portion of the truss to the left of ZW, considered as a solid rigid body. These forces are in equilibrium, and, since there are only three unknown forces, CE, EF, and FG, it is possible to solve for them by means of the three analytical equations of equilibrium, $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma M = 0$.

$$\Sigma Y = Y_{AK} + Y_{KL} + Y_{LG} + Y_{GF} + Y_{FE} + Y_{EC} + Y_{CB} + Y_{BA} = 0.$$

$$350 - 100 - 100 + 0 + FE \sin 60^\circ + 0 - 100 - 100 = 0.$$

$$350 - 400 + FE \frac{\sqrt{3}}{2} = 0.$$

$$FE = \frac{100}{\sqrt{3}}$$

From the positive sign of the result, the Y_{FE} must be positive.

The member FE is, therefore, in tension $\frac{100}{\sqrt{3}}$ pounds.

Take moments about the intersection of CE and EF at the point O.

$$\Sigma M = M_{AK} + M_{KL} + M_{LG} + M_{GF} + M_{FE} + M_{EC} + M_{CB} + M_{BA} = 0.$$

$$350.25 - 100.15 - 100.5 + GF.5\sqrt{3} + 0 + 0 - 100.10 - 100.20 = 0.$$

$$350.25 - 100(15 + 5 + 10 + 20) + GF.5\sqrt{3} = 0.$$

$$GF = -\frac{750}{\sqrt{3}}$$

From the negative sign of the result the moment of GF about O must be negative. GF, therefore, acts away from the section:

i.e., the member GF is in tension $\frac{750}{\sqrt{3}}$ pounds.

$$\Sigma X = X_{AK} + X_{KL} + X_{LG} + X_{GF} + X_{FE} + X_{EC} + X_{CB} + X_{BA} = 0.$$

$$0 + 0 + 0 + GF + FE.\cos 60 + CE + 0 + 0 = 0.$$

$$\frac{750}{\sqrt{3}} + \frac{100}{\sqrt{3}}.\frac{1}{2} + CE = 0.$$

$$CE = -\frac{800}{\sqrt{3}}.$$

From the negative sign of this result, the X_{CE} must be negative. CE, therefore, acts against the section;

i.e., the member CE is in compression $\frac{800}{\sqrt{3}}$ pounds.

The student is advised to choose other sections in the same truss and work out the stresses in the various members by this means.

CHAPTER VIII.

THE BEAM.

In the following discussion, the case of a beam supported freely on two abutments, one at each extremity of the beam, will be considered. This type of beam is the one most commonly met with in engineering practice, although cases do arise in which a beam structure is supported upon more than one abutment, or in which the abutment supports are not free, as in the case of a beam having its extremities built into the supporting walls. It is sufficient to point out that these last cases cannot be investigated by the theory in these pages alone, but necessitate a knowledge of the laws of the resistance of materials and of the calculus.

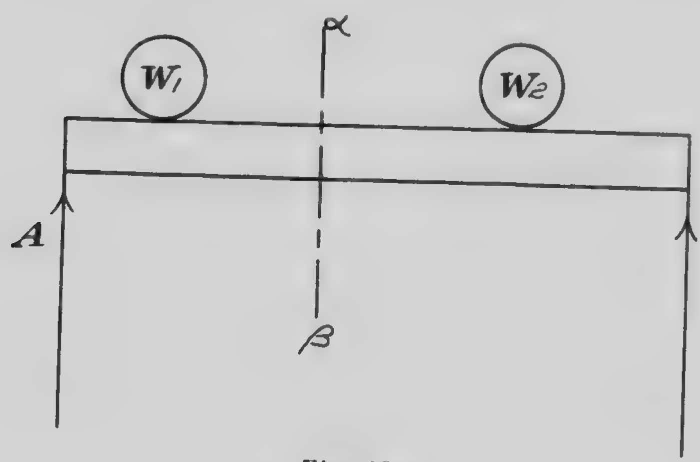


Fig. 35.

Let Fig. 35 represent a beam supported freely on two abutments and carrying any two loads, W_1 and W_2. The left-hand abutment reaction may be represented in magnitude by A, whose numerical value will depend entirely upon the distances of W_1 and W_2 from the abutment. Treat the portion of the beam to the left of the plane $\alpha\beta$ by the Method of Sections. The only **known** forces acting on this portion to the left of $\alpha\beta$ are the reaction A and the load W_1. These forces cannot by themselves maintain equilibrium. It is evident, therefore, that some other unknown force or forces must act

upon the portion of the beam being considered. The only possible place where such a force or forces could act is at the section $\alpha\beta$ in the beam. Let Fig. 36 represent the condition of affairs as far as the forces acting upon the body to the left of $\alpha\beta$ are concerned. These forces are in equilibrium; therefore, ΣX, ΣY, and ΣM must each equal zero.

$$\Sigma Y = Y_A + Y_{W1} + (\text{Y's of unknown forces}) = 0.$$
$$A - W_1 + (\text{Y's of unknown forces}) = 0.$$

Replacing the Y's of the unknown forces by their resultant, which may be denoted by V

$$A - W_1 + V = 0.$$
$$V = -(A - W_1).$$

From which it is seen that V is equal in magnitude but acts in the opposite sense to the algebraic sum of

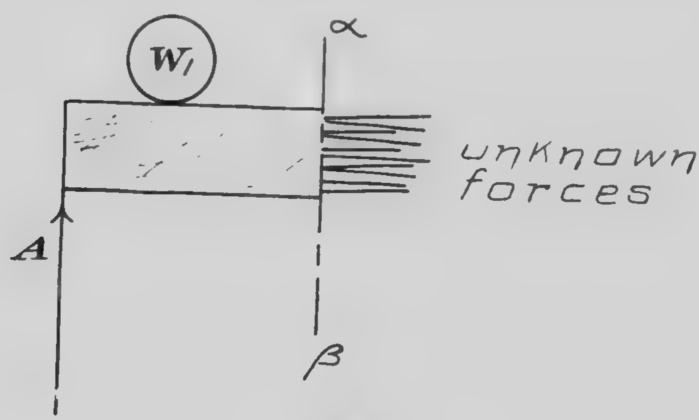


Fig. 36.

the vertical resolved parts of the exterior forces to the left of $\alpha\beta$; i.e., V resists $A - W_1$. This quantity $(A - W_1)$ is called the **Vertical Shearing Force** at the section under consideration.

Definition:—

The Vertical Shearing Force at any section is equal to the algebraic sum of the vertical resolved parts of the exterior forces acting on the body to the left of the section being considered.

This is a general definition, but since the loads and abutment reactions in most cases, and certainly in all

cases that will be taken up here, act in the vertical direction, a more concise definition may be employed.

The Vertical Shearing Force for Vertical Loading and Abutment Reactions is equal to the algebraic sum of all exterior forces to the left of the section being considered.

Applying ΣX to the forces acting as indicated in Fig. 36

$$\Sigma X = X_A + X_{W_1} + (X\text{'s of unknown forces}) = 0.$$
$$0 + 0 + (X\text{'s of unknown forces}) = 0.$$
$$(X\text{'s of unknown forces}) = 0.$$

Now, there are two ways in which the algebraic sum of the X's of the unknown forces may be equal to zero. Either all the X's are equal to zero, or the sum of the positive X's is equal in magnitude to the sum of the negative X's.

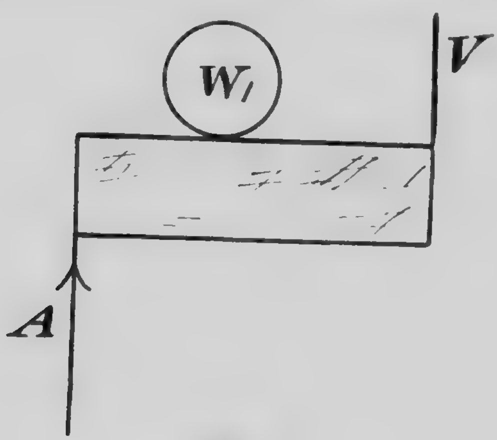


Fig. 37.

In order to determine which of these two cases is the true one, consider the forces acting on the portion of the body to the left of $\alpha\beta$ as they have been deduced so far. Let W_1 A; then V must act downward, for V has been shown to act contrary to $(A - W_1)$, which in this case will be a positive quantity; i.e., V acts with negative sense.

Fig. 37 represents the forces A, W_1, and V acting on the body. It is seen that these forces by themselves cannot give equilibrium; therefore, some other forces must act on the body. It is evident, then, that the X's

of the unknown forces are not all equal to zero, for if they were, the only other force which could exist would be a vertical force. But all the vertical forces possible are represented in the diagram. The positive X's must, then, equal the negative X's. Replace the positive and negative X's by their resultants, T and C, respectively. The true condition of affairs must be as represented in Fig. 38.

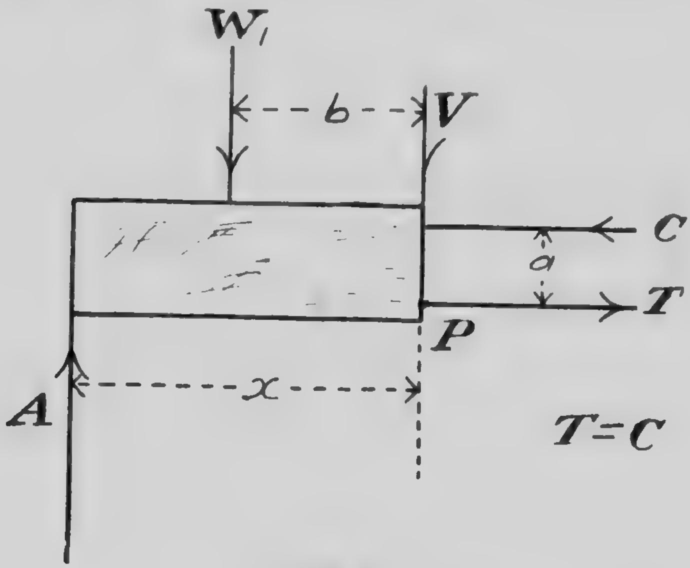


Fig. 38.

C and T form a couple of magnitude C.a. Apply ΣM to the forces acting on the body. Take moments about the point P.

$$\Sigma M = M_A - M_{W_1} + M_V + M_C + M_T = 0.$$

$$\text{But } M_V \text{ and } M_T = 0.$$

$$\text{Therefore, } M_C = -(M_A + M_{W_1}).$$

But $M_C = C.a$ = moment of couple formed by T and C; that is, the moment of this couple is equal in magnitude but opposite in sense to the algebraic sum of the moments of the exterior vertical forces to the left of $\alpha\beta$ about a point in the section.

In this particular case, which is perfectly general, it is seen that M_C resists $(M_A + M_W)$. This quantity $(M_A + M_W)$ is called the **Bending Moment** at the section considered.

Definition: -

The Bending Moment at any section for vertical loading and abutment reactions is equal in magnitude to the algebraic sum of the moments of the exterior forces to the left of the section, about a point in the section.

CHAPTER IX.

VERTICAL SHEARING FORCE AND BENDING MOMENT FOR STATIONARY LOADS ON A BEAM.

The student is particularly advised to memorize the definitions of the Vertical Shearing Force (V.S.F.) and the Bending Moment (B.M.) at any section in a beam as given in the last chapter.

In explanation of the meaning of these terms, V.S.F. and B.M., it may be pointed out that in a beam under a load there is a tendency to bend; and because the deflection due to bending is different at various sections, it is reasonable to suppose that the bending influence is variable for different sections of the beam, which reasoning is borne out by the theory. The B.M.

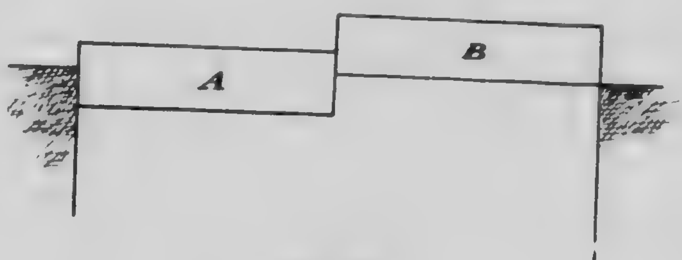


Fig. 39.

is related to this bending influence. Secondly, any portion of a beam has a tendency to slip past, or **shear** past, the other portion of the beam. For instance, in Fig. 39 the portion A has sheared past B. Whether any given portion of a beam actually does shear or not is aside from the question that there exists such a tendency. The V.S.F. is related to this tendency, which is variable for different sections.

To Trace the Variation in V.S.F. Due to a Single Concentrated Load.

In Fig. 40, consider **any** section between the load W and the left-hand abutment. The V.S.F. at any such section = algebraic sum of exterior forces to left of section.

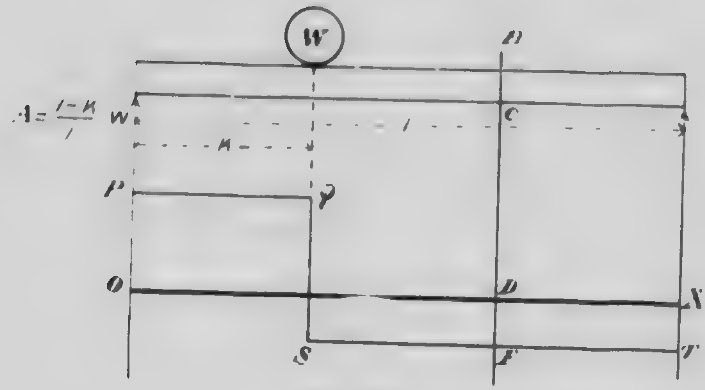


Fig. 40.

The only force to the left of such a section is the abutment reaction $A = \frac{l - K}{l}.W$.

Therefore, V.S.F. for all sections between W and A, = A

= a positive constant value.

Consider **any** section to the right of W.

V.S.F. at any such section = algebraic sum of exterior forces to left of section.

The algebraic sum of the forces to left of any such section is (A — W).

Therefore, V.S.F. for all sections to right of W (A — W), and since W > A —

V.S.F. = a negative constant value.

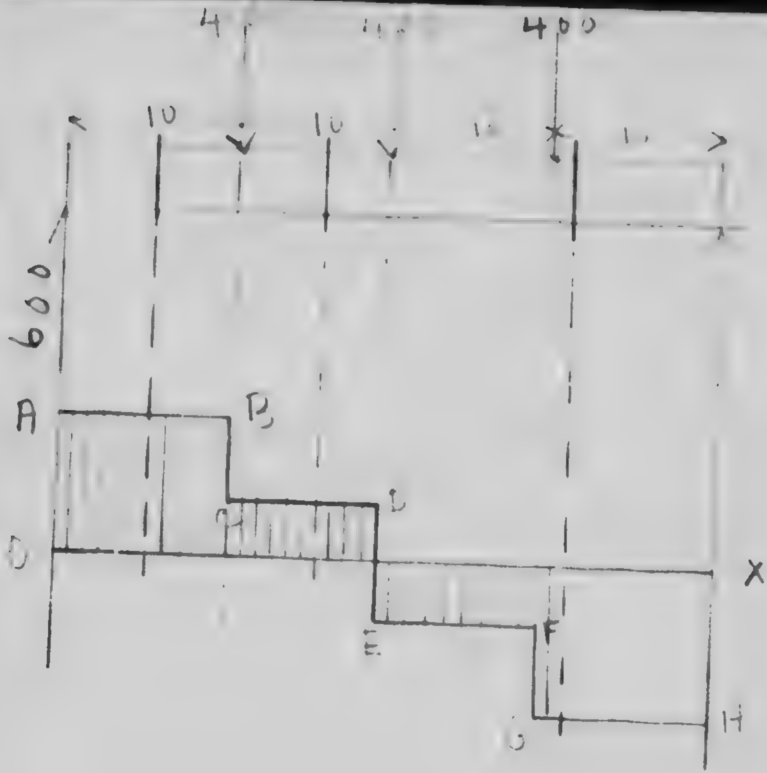
Let OX be drawn parallel to the longitudinal axis of the beam.

Let y = V.S.F. at any section of the bean.

For all sections to the left of W, y = positive constant = A.

For all sections to the right of W, y = negative constant = (A — W).

Plotting these ordinates with OX as a base line, and plotting them so that the value of the shear at any section lies directly under that section, the curves will be of the form PQ and ST, two straight lines parallel to OX. The shear at a section immediately underneath W is indeterminate, and may be represented as such by joining Q and S. The diagram PQST is known as the **V.S.F. Diagram.** If it is desired to find the V.S.F. at any section such as BC, all that is necessary is to pro-


400
10
600
A
B
E
F
H
X

duce BC to intersect OX and the locus ST at D and E respectively. The ordinate DE represents the V.S.F. at section BC.

To Trace the Variation in V.S.F. Due to any Number of Concentrated Loads.

In this case an actual numerical example will serve the purpose as well as, if not better than, mere general values.

Consider the case of a beam 50 feet long supporting four concentrated loads of magnitude, 100, 200, 50, and 100 pounds, situated at distances 10, 20, 25, and 30 feet, respectively, from the left-hand abutment. Required to draw the V.S.F. diagram.

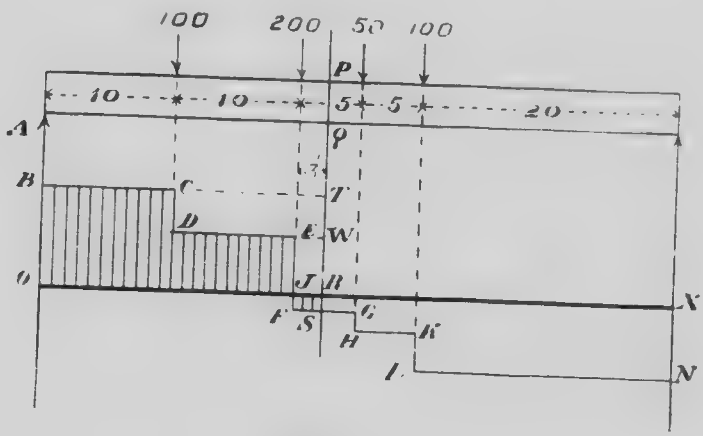


Fig. 41.

In Fig. 41 the left-hand abutment reaction

$$A = \tfrac{4}{5}\cdot 100 + \tfrac{3}{5}\cdot 200 + \tfrac{1}{2}\cdot 50 + \tfrac{2}{5}\cdot 100,$$
$$= 265 \text{ pounds.}$$

Let y = V.S.F. at any section.

Between A and first load, $y = A = 265$.

Between first and second load, $y = 265 - 100 = 165$.

Between second and third load,

$$y = 265 - 100 - 200 = -35.$$

Between third and fourth load,

$$y = 265 - 100 - 200 - 50 = -85.$$

Between fourth load and right-hand abutment,

$$y = 265 - 100 - 200 - 50 - 100 = -185.$$

Plotting these ordinates with OX as base line, a V.S.F. diagram of the form BC, DE, FG, HK, LN is obtained, such that the ordinate to the diagram immediately beneath any section of the beam represents the value of the V.S.F. at that section.

It will be noticed that **the change in value of shearing force** from a section immediately to the left of any load to a section just to the right of the load **is equal to the magnitude of the load itself.** For instance, in the last numerical case, the changes represented by CD, EF, GH, and KL are each equal in magnitude to the loads immediately above these lines.

Representation of Bending Moment at any Section of a Beam.

Let PQ (Fig. 41) be the section, the B.M. at which it is required to represent. PQ produced intersects the V.S.F. diagram in RS.

Let PQ be three feet to the right of the 200-pound load.

Produce BC and DE to intersect PQ at T and W.

B.M. at PQ = A × 23 — 100 × 13 — 200 × 3.

But A × 23 = OB × BT = area OT.

100 × 13 = CD × CT = area DT.

200 × 3 = EF × EW = area FW.

Therefore, B.M. = area OT — area DT — area FW.

= area OBCDEJ — area JFSR.

= area of V.S.F. diagram to left of PQ.

It must, of course, be remembered that areas above OX are considered positive and those below OX negative. Keeping this last statement in mind, it is seen that

The area of the V.S.F. diagram to the left of any section represents the B.M. at that section.

The B.M. at any section is always positive, for the positive area of the V.S.F. diagram to the left of any section is, with one exception, always greater than the negative area to the left of the same section. It may be proven geometrically that the positive and negative areas to the left of the right-hand abutment are equal; hence, the B.M. at this abutment is zero. The B.M. at the left-hand abutment is also zero, since there is no area of the V.S.F. diagram to the left of that section.

Section at which Maximum B.M. Occurs for a Single Load.

ABDF (Fig. 42) is the V.S.F. diagram for the load W. Consider areas above OE positive, below OE negative.

The B.M. at any section NP to the left of the load is represented by the area ANPO.

The B.M. at any section GL, to the right of the load, is represented by the area ABCO — CGLD.

Both of these areas are less than the positive area ABCO. Therefore, the area ABCO must represent the greatest B.M.

But ABCO represents the B.M. at a section infinitely close to BC, but still to the left of the load. Practically, this is the section BC. It follows that:

The Maximum B.M. for a single load in a given position occurs at a section directly under the load.

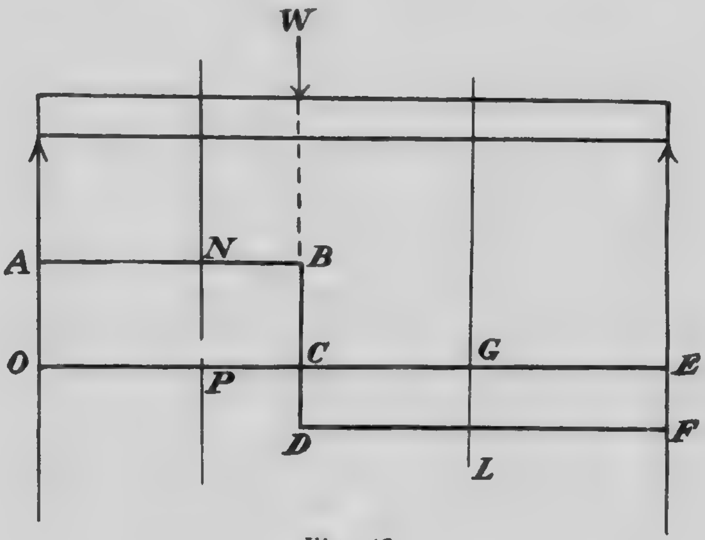


Fig. 42.

Uniformly Distributed Load.

A uniformly distributed load is one such that, no matter how small a portion of the area loaded is considered, the load per **unit area** is always the same.

Since the discussion is confined to co-planar forces, a uniformly distributed load in the following problems will mean a load of so much per **unit length**.

It may be pointed out that in practice the term "uniformly distributed load" is very often applied to a number of concentrated loads placed very close to one another. For instance, a crowd of people standing on a floor is referred to as a uniformly distributed load of so much per square foot, when in reality the load is made up of a number of concentrated loads placed very close to one another. However, the assumption gives results well within all limits of safety.

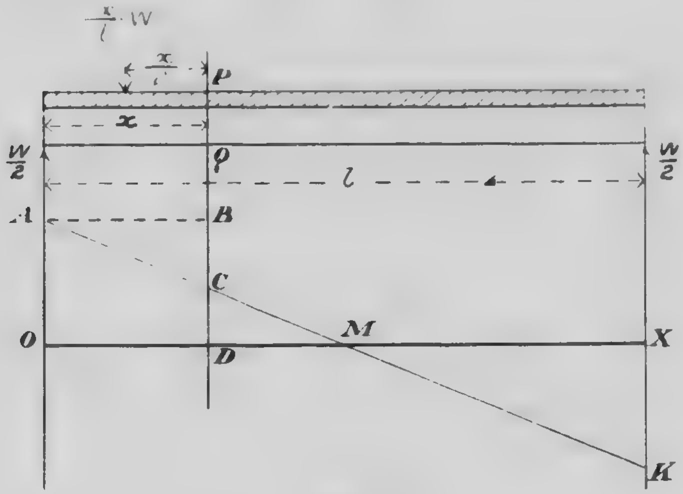


Fig. 43.

To Trace the Variation of V.S.F. and B.M. in a Beam Supporting a Uniformly Distributed Load of W Pounds.

Let the hatched area of Fig. 43 represent the load uniformly distributed over the entire length of the beam.

$$\text{Total load} = W \text{ lbs.} \quad \text{Abutment reactions} = \frac{W}{2}.$$

$$\text{Load per unit length} = \frac{W}{l}.$$

Take **any** section PQ at a distance x from the left-hand abutment.

V.S.F. at PQ = y = (left-hand abutment reaction) − (load to left of PQ).

$$\text{Load to left of PQ} = \frac{W}{l} \cdot x.$$

Therefore, $y = \frac{W}{2} - \frac{W}{l}.x$

If the values of y be plotted with OX as axis of x, the curve will be a straight line such that, as in the previous cases, the value of the V.S.F. at any section will be directly under that section. This is shown by the line AK.

To trace the curve, $y = \frac{W}{2} - \frac{W}{l}.x$ .... (1.)

$$\text{When } x = 0,\ y = \frac{W}{2} = OA.$$

$$\text{When } x = l,\ y = -\frac{W}{2} = XK.$$

$$\text{When } y = 0,\ x = \frac{l}{2} = OM.$$

From this it is seen that the V.S.F. varies from a positive value of $\frac{W}{2}$ at the left-hand abutment, to a value $-\frac{W}{2}$ at the right-hand abutment, being equal to zero at the centre of the beam.

In order to consider the variation in B.M., let the load to the left of any section PQ be replaced by an equivalent concentrated load of magnitude $\frac{x}{l}.W$, which will act a distance $\frac{x}{2}$ from PQ as dotted in Fig. 43.

B.M. at PQ $= \frac{W}{2}.x - \frac{W}{l}.x\frac{x}{2}$ .... (2.)

Produce PQ to intersect the V.S.F. diagram at C and D. Draw AB perpendicular to PQ.

OD and DC are co-ordinates to a point on AK below PQ.

$$OD = x;\ y = DC.$$

Therefore, from (1.), $y = DC = \frac{W}{2} - \frac{W}{l}.x \ldots (3.)$

From the figure it is seen that

$CB = DB - DC = OA - DC$ .... (4.)

$$\text{But } OA = \frac{W}{2}.$$

Substitute in (4.) this value of OA, and also the value of DC found in (3.).

$$\text{Then, } CB = \frac{W}{2} - \left(\frac{W}{2} - \frac{W}{l}.x\right).$$

$$\frac{W}{l}.x.$$

Tabulating these results—

$$\frac{W}{2} = OA.$$

$$x = OD = AB$$

$$\frac{W}{l}.x = CB.$$

Substituting these values into (2.).

$$B.M. = OA \times OD - CB \times \frac{AB}{2}.$$

= Area OB — Δ ABC.
= Area OACD.
= Area of V.S.F. diagram to left of section.

The B.M. evidently varies from a value of zero at the left-hand abutment to zero at the right-hand abutment, having always positive values.

It is seen that **the maximum B.M. for a beam supporting a uniformly distributed load over its entire length is at the centre of the beam,** for the area AMO is the greatest positive area.

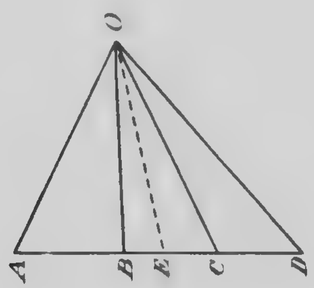


Fig. 44 b.

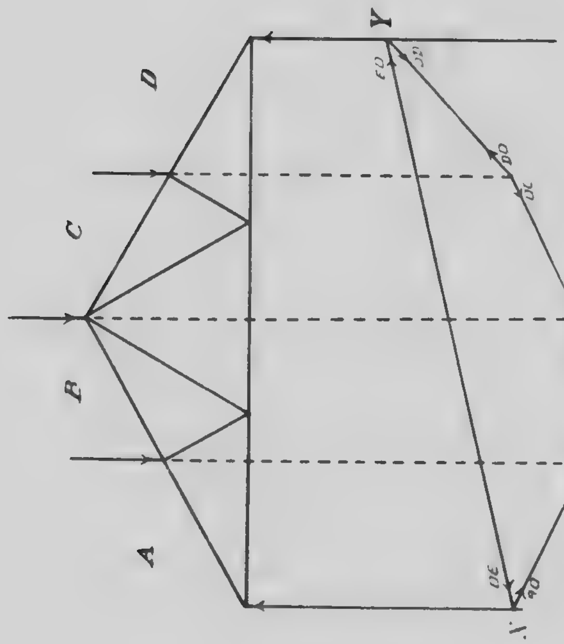


Fig. 44 a.

Graphical Determination of Abutment Reactions.

In the work previous to this, the abutment reactions whenever desired have been calculated analytically. There is, however, a graphical method of determining these reactions, the procedure being as follows:

Let Fig. 44 a represent any frame structure resting freely on two abutments and supporting the loads AB, BC, CD of any magnitude whatever. These loads, together with the unknown abutment reactions DE and EA, give equilibrium.

The vector polygon, AB, BC, and CD (Fig. 44 b), may be drawn to represent the three loads, and, since the forces acting on the structure are in equilibrium, the vector polygon must close. The only forces that can close it are the two abutment reactions. It follows, then, that between D and A there lies some point E, such that DE and EA represent these two reactions. If E can be located, the problem is solved.

Choose any pole O in Fig. 44 b and join O to A, B, C, and D. As far as possible construct the funicular polygon for the forces acting on the structure, commencing by balancing AB by BO and OA. Proceed by balancing BC with CO and OB, and CD by DO and OC. Produce the lines of action of OA and DO to intersect the abutment reactions at X and Y respectively. Now, no matter where the point E lies between D and A in Fig. 44 b, DE and EA will be balanced by EO, OD, and AO, OE, respectively. If the abutment reactions were balanced at X and Y by these forces, it is seen that the only possible manner in which EO and OE, which are as yet unknown, can act in order that the funicular polygon may close, is in the line XY. This determines the point E, for, in order that the forces EO and OE may act in the direction XY, the line joining O to E must be parallel to XY; i.e., **to locate E, through O draw OE parallel to XY, intersecting DA at E.** DE and EA (Fig. 44 b) represent the required reactions.

Relation between Bending Moment and the Funicular Polygon.

Let Fig. 45 represent a beam supporting any general loading, AB, BC, and CD. To the right of the figure is the vector polygon, AB, BC, CD, DE, and EA, the abutment reactions DE and EA having been found

graphically by means of the funicular polygon, as explained in the last problem. It is required to find the value of the B.M. at any section (1.).

Produce (1.) to intersect the funicular polygon at G and K. Let the perpendicular distance of the pole O from AB be Z.

The B.M. at (1.) = Algebraic sum of the moments of EA and AB about any point in (1.).
= Moment of the resultant of EA and AB about any point in (1.).

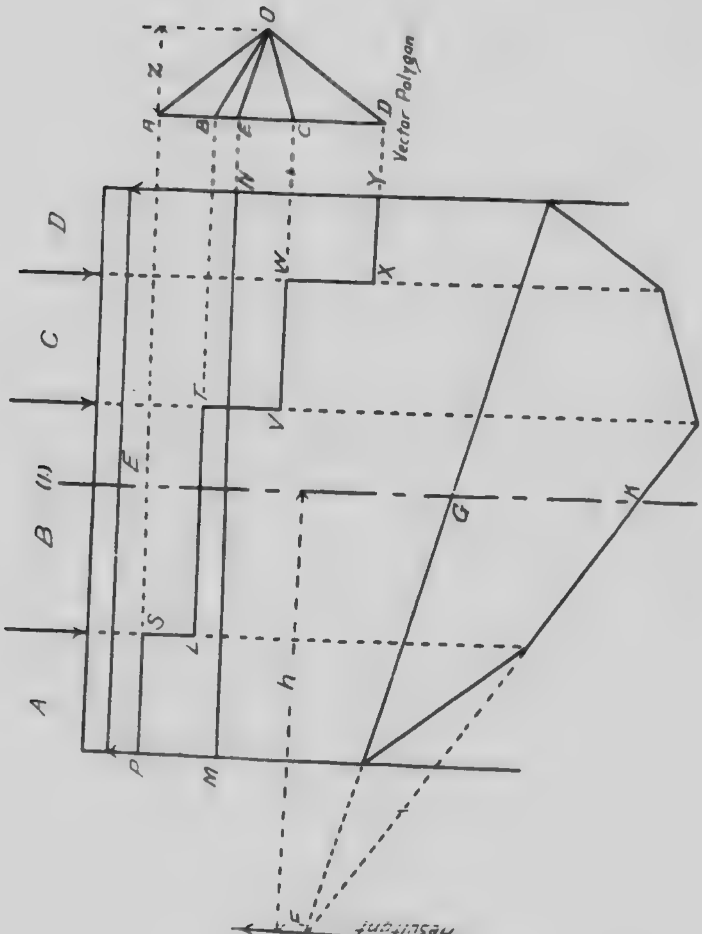


Fig. 45.

To locate the resultant of EA and AB; i.e., the resultant of all forces to the left of plane (1.):

In discussing the properties of the Equilibrium or Funicular Polygon, it was shown that if a section be taken through two members of the frame, the resultant of the external forces to either side of the section acts through the intersection of the cut sides of the polygon. Section (1.) cuts two sides of the polygon at G and K. If, then, the two cut sides be produced to intersect at F, as indicated in Fig. 45, it is evident that the resultant of the forces EA and AB must act through this point. But from the Vector Polygon it is seen that the resultant of EA and AB is represented by EB. Drawing through F a line parallel to EB, the true position of the required resultant is found. The moment of this resultant about a point in section (1.) will give the desired B.M. Let h be the distance of this resultant from section (1.).

Now, FG EO.
FK OB.
GK BE.

Therefore, Δ's FGK and BEO are similar.

Then, $$\frac{EB}{GK} = \frac{\text{altitude } z.}{\text{altitude } h.}$$

Or, $EB \times h = GK \times z$ .... (3.)

But EB represents the resultant of the forces to the left of section (1.).

Therefore, $EB \times h = B.M.$ .... (4.)
i.e., from (3.) $B.M. = GK \times z$................. (5.)

Now, GK is called the ordinate of the funicular polygon at section (1.), and it is seen that the B.M. is represented by this ordinate multiplied by a constant quantity z; so that, **provided the proper scale is used, the B.M. at any section is represented by the ordinate of the funicular polygon immediately beneath the section.**

To Find the Scale of B.M.'s to be Used with Ordinates of the Funicular Polygon.

From (4.) in the last discussion—

B.M. = EB × h.

(EB in inches × scale of forces) (h in inches × scale of dimensions).

EB × h (scale of forces × scale of dimensions).

But from (3.) EB × h = GK × z.

Therefore, B.M. = GK (z × scale of forces × scale of dimensions); i.e., the scale to use on GK in order to find the actual value of the B.M. is **(z × scale of forces × scale of dimensions),** z being taken in inches if the scales are per inch.

Caution:

In order to get a good working scale of B.M.'s, the pole O should be so chosen with reference to AD, that z will be a quantity which will, when multiplied by the scales of forces and dimensions, give an even quantity suitable for use with ordinary scales.

A Graphical Method of Constructing the V.S.F. Diagram for Concentrated Loads.

Let the vector diagram of the forces acting on the beam be drawn to one side and slightly below the representation of the beam as shown in Fig. 45. It will be assumed that the abutment reactions DE and EA have been found. From E draw a line ENM parallel to the longitudinal axis of the beam, and intersecting the abutment reactions at N and M.

The V.S.F. at any section between the left-hand abutment and the first load is equal to the left-hand abutment reaction, the value of which reaction is represented by EA. Therefore, if the scale of shearing forces be chosen the same as the scale of forces used in constructing the vector diagram, the ordinates from MN to the line PS, which is drawn parallel to MN through A, will represent the V.S.F. values at any section between the left-hand abutment and the first load, for these ordinates will each be equal to EA, which is the value of the V.S.F. between the sections indicated.

The V.S.F. for any section between the first and second load is equal to the left-hand abutment reaction minus the first load AB. Graphically, this is represented by

$$EA - AB = EB.$$

Therefore, LT, drawn parallel to MN through B, gives a diagram, the ordinates from MN to which, represent the V.S.F. values between the first and second load.

Following the same line of reasoning, the ordinates from MN to VW and XY, which are drawn parallel to MN from C and D, respectively, represent the V.S.F. values from the second load to the right-hand abutment.

CHAPTER X.

V.S.F. AND B.M. FOR MOVING LOADS ON A BEAM.

To Trace the Variation in V.S.F. and B.M. for a Single Moving Load.

Let a load of W pounds travel over the beam to a position distant x from the left-hand abutment as represented in Fig. 46. The left-hand abutment reaction A will then be $\frac{l - x}{l}.W$.

RBDH is the V.S.F. diagram for the load in this position.

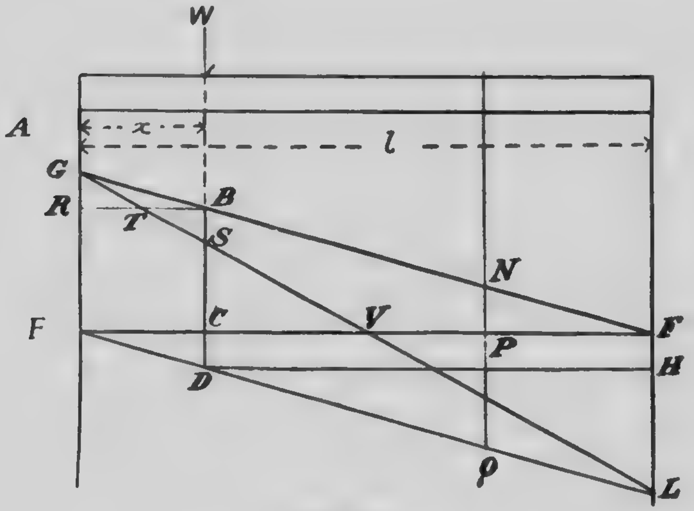


Fig. 46.

Now, since the load is a moving one, it would be extremely inconvenient to draw a series of V.S.F. diagrams for many or all positions of the load.

Looking at the diagram RBDH, which is a perfectly general one, it is seen that because RB and DH are parallel to EF, BC represents the value of the V.S.F. at any plane to the left of the load and CD the V.S.F. at any plane to the right of the load.

But V.S.F. at any plane' to left of load $= A = \frac{l-x}{l}.W = BC$; and V.S.F. at any plane to right of load $= A - W = \frac{l-x}{l}.W - W = CD$.

If, then, ordinates $y = \frac{l-x}{l}.W$ be plotted underneath the load as it moves to different positions, such that y represents the value of the V.S.F. at any plane to the left of the load, and ordinates $y' = \frac{l-x}{l}.W - W$ be also plotted underneath the load in the same series of positions, so that y' represents the V.S.F. at all planes to the right of the load, the locii of the points so obtained will be the straight lines GF and EL, respectively. (y' will, of course, be negative for $W > \frac{l-x}{l}.W$.)

GFLE is a V.S.F. diagram for the moving load W. Supposing it is desired to find the V.S.F. at a plane NQ: (1.) when the load is over BD; (2.) when the load is over NQ.

(1.) The V.S.F. at NQ when the load is over BD is represented by CD, for CD represents the V.S.F. for **all** planes to right of load when load is over BD.

(2.) When the load is directly over NQ, the V.S.F. at NQ is indeterminate, but NP represents the V.S.F. at all planes to left of NQ, and PQ represents the V.S.F. at all planes to right of NQ. These results may be arrived at analytically by letting x be the distance of the load from the left-hand abutment.

For any plane to the left of the load—

$$\text{V.S.F.} = y = \frac{l-x}{l}.W.$$

For any plane to the right of the load—

$$\text{V.S.F.} = y' = \frac{l-x}{l}.W - W.$$

To Find the Maximum Possible B.M. as the Load Moves Over Beam.

For the load in a given position it has been shown that the maximum B.M. is at a section directly under the load. It is only necessary to compare these maximum B.M.'s for the load in different positions in order to find the greatest possible B.M.

Taking the load in a position represented in Fig. 46, the maximum B.M. for this position is represented by RBCE.

Join GL.

The area GSCE is then equal to the area RBCE, for the area RTSCE is common to both, and the Δ GRT equals the Δ TBS.

Therefore, the area GSCE represents the maximum B.M. for the load in that position. In the same way, for the load in other positions, the maximum B.M. is represented by the area between GL and EF to the left of the section under the load.

It is seen that the greatest B.M. that will occur is represented by the positive area GVE. This area represents the B.M. when the load is at the centre of the beam.

Hence:

The maximum possible B.M. for a single moving load occurs directly under the load when it is at the centre of the beam.

CHAPTER XI.

PULLEY SYSTEMS.

It is the intention, in the following discussion, to find out how one or more pulley blocks may be most advantageously arranged in order to lift or move a given weight.

In most of the problems the ropes of the system are assumed to be parallel to one another. This, of course, is not quite true in practice, but the error arising from such an assumption is so slight that it may be neglected, together with the errors that arise from assuming that the sheaves have frictionless bearings and that their weights are negligible.

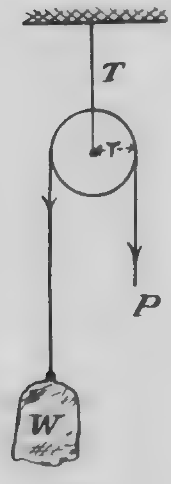


Fig. 47.

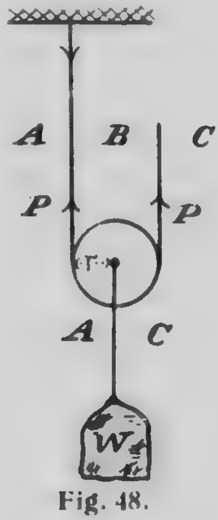


Fig. 48.

Mechanical Advantage.

The mechanical advantage of a system of pulleys is the ratio of the weight lifted to the force exerted on the free end of the rope.

The mechanical advantage is sometimes referred to as the **efficiency of the system.**

I. Let Fig. 47 represent a single block fastened to some support. At one end of the rope is fastened a weight, W. It is required to find the force P which must be exerted at the free end of the rope in order to keep

the weight W in equilibrium. Any greater force than P will lift W.

Consider the pulley as a rigid body. It is acted upon by three forces, P, W, and T, in equilibrium.

Take moments about the centre of rotation of the sheave.

$$\begin{aligned} \Sigma M = M_P + M_W + M_T &= 0. \\ P.r - W.r + 0 &= 0. \\ P &= W; \end{aligned}$$

i.e., with such an arrangement of the block the force exerted at the free end of the rope to keep the system in equilibrium must be equal to the weight.

$$\text{Mechanical Advantage} = \frac{W}{P} = 1.$$

II. Let the block be attached to the weight and one end of the rope made fast to some abutment as represented in Fig. 48.

Consider the pulley as a rigid body. It is acted upon by the three forces AB, BC, and CA (using Bow's Notation) in equilibrium.

Take moments about the axis of rotation as in I.

$$\begin{aligned} \Sigma M = M_{AB} + M_{BC} + M_{CA} &= 0. \\ AB.r - BC.r + 0 &= 0. \\ AB &= BC. \end{aligned}$$

If a force of P pounds be applied at the free end of the rope, then BC = P = AB.

To find the magnitude of P—

$$\begin{aligned} \Sigma Y = Y_{AB} + Y_{BC} + Y_{CA} &= 0. \\ P + P - W &= 0. \\ P &= \tfrac{1}{2} W. \end{aligned}$$

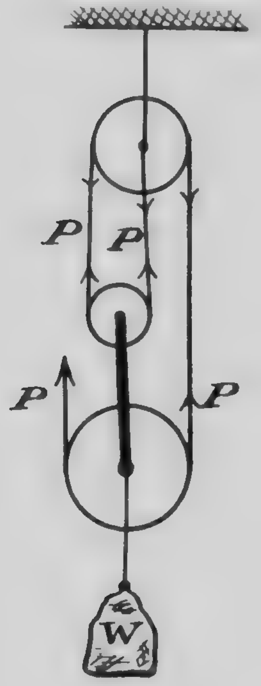
$$\text{Mechanical Advantage} = \frac{W}{P} = \frac{W}{\frac{1}{2}W} = 2.$$

III. In a system such as represented by Fig. 49, if a force P be applied to the free end of the rope, there will exist a tension of P pounds throughout the rope system.

The two lower blocks are connected together rigidly. They may, then, be considered as a body acted upon by four upward forces of magnitude P, due to the tension in the ropes, and a fifth downward force of magnitude W. These forces are in equilibrium.

$$\Sigma Y \text{ for lower two blocks} = 4P - W = 0.$$
$$P = \tfrac{1}{4} W.$$

$$\text{Mechanical Advantage} = \frac{W}{P} = 4.$$



No. 49.

The following cases, it is easily seen, are merely special examples of the preceding problems. For this reason the solution of each case is condensed.

The inverse arrangement of the last case does not give as good efficiency. This system is shown at Fig. 50.

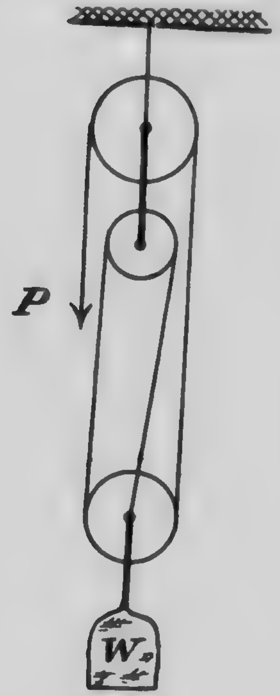


Fig. 50.

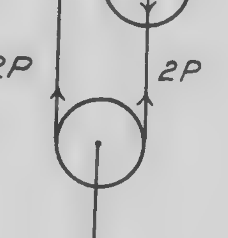


Fig. 51.

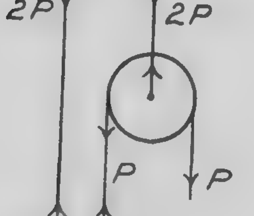


Fig. 52.

$$\Sigma Y \text{ for lower block} = 3\,P - W = 0.$$
$$P = \tfrac{1}{3}\,W.$$
$$\text{Mechanical Advantage} = \frac{W}{P} = 3.$$

Referring to Fig. 51, if P be the magnitude of the force applied to the free end of the rope, then the tension in the rope passing over the lower pulley will be 2 P as indicated.

$$\Sigma Y \text{ for lower block} = 2\,P + 2\,P - W = 0.$$
$$P = \tfrac{1}{4}\,W.$$
$$\text{Mechanical Advantage} = \frac{W}{P} = 4.$$

In an arrangement such as represented by Fig. 52, if P pounds be applied to the free end of the rope, the mechanical advantage will be 3. The student is advised to figure this out for himself.

The Weston Differential Pulley.

In the ordinary pulley systems, where there is a free rope end, there is always more or less inconvenience arising from the large amount of rope lying around. Added to this is the fact that in order to lift a weight of any size, a number of blocks must be used. To obviate all this, the Differential Pulley is used. Fig. 53 illustrates diagrammatically the simplest form of this type of pulley, upon which is based all other differential arrangements of blocks. This type is known as the Weston Differential Pulley. The system consists of an upper block composed of two sheaves of different radii, cast integral, and a lower block, whose radius is generally about that of the smaller of the upper two, although this is immaterial. The upper two blocks have roughened rims to prevent the endless chain, which passes over the system, from slipping. There is always some arrangement cast on the upper block casing which allows the chain to run when pulled either way, but if left hanging idle, the chain checks. Thus it is possible to lift or lower a weight to any desired height and leave it unattended. The lower block has a smooth circumference.

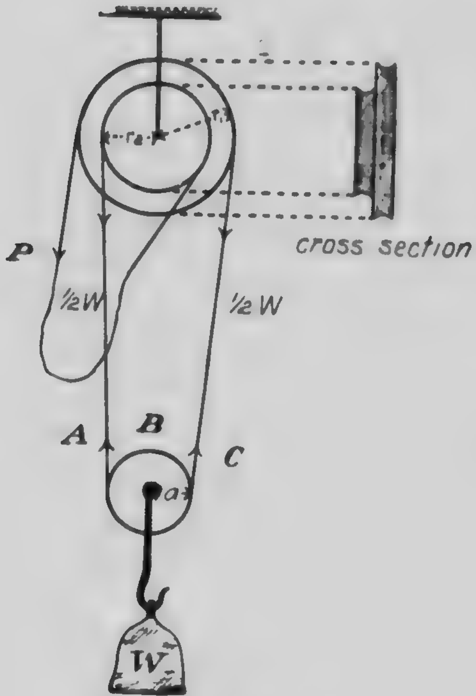


Fig. 53.

For lower block, take moments about axis of rotation.

$$\Sigma M = M_{AB} + M_{BC} + M_{W} = 0.$$
$$AB.a - BC.a + 0 = 0.$$
$$AB = BC \ldots\ldots\ldots\ldots(1.)$$
$$\Sigma Y = Y_{AB} + Y_{BC} + Y_{W} = 0.$$
$$AB + BC - W = 0\ldots\ \ldots\ldots\ldots(2.)$$

Substitute for AB from (1.) into (2.)

$$2\ BC = W.$$
$$BC = AB = \frac{W}{2}.$$

Let a force of P pounds be applied to free end of chain.

For the upper block, taking moments about axis of rotation:—

$$\Sigma M = P.r_1 + \frac{W}{2}.r_2 - \frac{W}{2}.r_1 = 0.$$

$$P = \frac{W(r_1 - r_2)}{2 \quad r_1}.$$

$$\text{Mechanical Advantage} = \frac{W}{P} = \frac{2r_1}{r_1 - r_2}.$$

The Mechanical Advantage of such a system is then equal to twice the radius of the large block divided by the difference between the radii of the large and small block.

To Design a Differential Pulley.

Design a differential pulley to lift a ton weight by applying ten pounds force to the free end of the system.

$$\text{Mechanical Advantage} = \frac{W}{P} = \frac{2000}{10} = \frac{2r_1}{r_1 - r_2}.$$

$$\text{Simplifying: } \frac{2000}{10} = \frac{2r_1}{r_1 - r_2}.$$

$$\frac{r_1}{r_2} = \frac{200}{198},$$

so that any differential pulley whose upper blocks have their radii in the ratio of 200:198 will satisfy the demands of the problem.

What would the Mechanical Advantage be if the radii of the two upper blocks of a differential pulley were equal?

Answer: Infinite. The key to this seeming paradox is that, although the advantage is infinite, it would take an infinite time to accomplish anything with such a pulley.

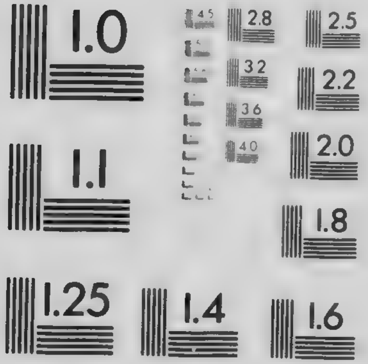
Important.

In the preceding discussion of pulley systems, it is seen that it is possible to lift a large weight by the application of very little force. The student must not

fall into the trap of thinking that by doing comparatively little **work** at one end of the system that a large amount of **work** can be done at the other end. The work done at both ends of the system is exactly the same in theory. Practically, there will be a mechanical loss due to friction, etc., between the free end and the weight being lifted.

MICROCOPY RESOLUTION TEST CHART

ANSI and ISO TEST CHART No. 2



APPLIED IMAGE Inc

CHAPTER XII.

FRICTION.

The term **Friction** is used to designate the resistance offered when we attempt to move, or actually do move, one body which rests in contact with another. It is a variable quantity, depending on circumstances, but usually reaches its maximum value when the body is just on the point of motion. The name **Limiting Friction** is used to denote this maximum value.

There are two kinds of friction, **Sliding** and **Rolling**, the former name being applied to cases where a body slides over another, and there is a definite plane of contact; the latter to cases where wheels, etc., roll along with small areas of contact.

One general law which applies to all kinds of friction states that—

Friction always acts in such a way as to oppose motion.

The following cases should be examined and the directions in which friction acts fully determined:—

1. Ladder resting against a wall.
2. Iron wheel rolling on an elastic plane.
3. Rubber-tired wheel rolling on an iron plane.
4. Rope with a weight on each end, one greater than the other, stretched over a fixed pulley. (a.) Rope just on the point of moving. (b.) Rope moving.
5. Rod resting on a table pulled at one end perpendicularly to its length.
6. Ring laid on a rough, horizontal table and pulled at any point tangential to ring so that ring is just on point of motion.

The laws of rolling friction have not been fully investigated, and will not be discussed here. Those of sliding friction are more fully understood, being based, however, on experiment, and being true only within certain limits.

The chief laws are:—

1. When motion is just about to take place, there is an approximately constant ratio between the friction and the **normal pressure** between the two bodies, which is called the Coefficient of Statical Friction.

This ratio is usually a small fraction, and tables are constructed empirically, giving values for different bodies.

2. When motion takes place, the coefficient is smaller than in the former case, and is termed the Coefficient of Dynamical Friction. This statement is true only for small velocities.

3. Friction depends upon the materials used and on their temperature.

Let Fig. 54 represent a body in equilibrium resting on an inclined plane, its weight W acting as indicated through the centre of gravity. There is also acting on the body the reaction, N, of the plane perpendicular to

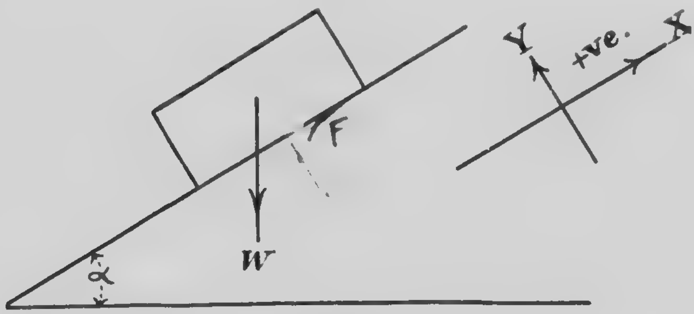


Fig. 54.

the surface of contact and the friction F. These forces acting on the body are in equilibrium. Therefore, $\Sigma X = 0$; $\Sigma Y = 0$; and $\Sigma M = 0$.

Taking the axis of X parallel to surface of plane as indicated—

$$\Sigma X = -W \cos (90 - \alpha) + F = 0.$$

$$F = W \sin \alpha.$$

$$\text{or } W = \frac{F}{\sin \alpha} \text{ .... } (1.)$$

$$\Sigma Y = -W \sin (90 - \alpha) + N = 0.$$

$$N = W \cos \alpha \text{...... } (2.)$$

Substitute value of W from (1.) into (2.).

$$N = \frac{F}{\sin \alpha}. \cos \alpha.$$

$$\text{or } \frac{F}{N} = \tan \alpha \text{......... } (3.)$$

Now, imagine the plane as being gradually tipped up so that the angle of inclination α becomes greater and greater. A position will be eventually reached beyond which the plane cannot be raised without causing the body to slip. The plane is then said to be inclined at the "**angle of repose.**"

Definition:—

If a body rest on a plane surface, the Angle of Repose is that angle beyond which the plane cannot be inclined without the body moving down the plane.

When the plane reaches the angle of repose from (3.)—

$$\frac{F}{N} = \text{tan. angle of repose.}$$

$\frac{F}{N}$, in this particular instance, is known as the **coefficient of friction,** usually designated by the Greek letter μ.

Coefficient of Friction $= \mu =$ tangent of angle of repose.

From the previous discussion it is seen that if a body rest on a plane surface which is inclined at an angle less than the angle of repose (sometimes called the angle of friction), the body will not slip down the plane. If now, some external force be applied to the body so as to cause the body to be **just on the point of moving** either up or down the plane, the resistance offered by friction can in every such case be experimentally shown to be of a magnitude given by

$$F = N.\mu \qquad (4.)$$

Where F = frictional resistance.
N = pressure normal to the plane.
μ = coefficient of friction for plane and body.

Problems.

A body weighing 100 pounds rests on a plane surface which is inclined to the horizontal at an angle of 30°, the angle of repose for the plane and body being 45°. Find what force acting parallel to the plane will

cause the body to be on the point of moving: (1.) up the plane; (2.) down the plane.

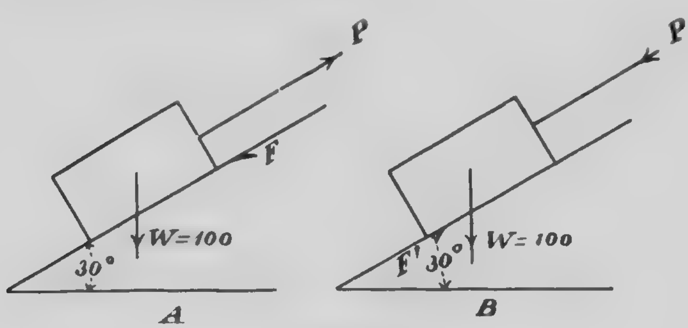


Fig. 55.

Fig. 55 A represents case (1.).

The forces known to act on the body are, W = 100 pounds acting through the centre of gravity of the body; F, the friction; P, the required force.

Since these forces are in equilibrium, $\Sigma X = 0$; $\Sigma Y = 0$; $\Sigma M = 0$.

$$\Sigma X = X_W + X_P + X_F = 0.$$

$$-W \cos 60° + P - F = 0 \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.)$$

Now $F = N.\mu$.

$$\text{Where } N = W \cos 30° = 100.\frac{\sqrt{3}}{2} = 50\sqrt{3}.$$

and μ = tan angle of repose = tan 45° = 1.

Therefore, $F = 50\sqrt{3} \times 1$.

Substituting this value of F into (5.),

$$-100.\tfrac{1}{2} + P - 50\sqrt{3} = 0.$$

$$P = (50\sqrt{3} + 50) \text{ pounds.}$$

Referring to Fig. 55 B for case (2.), P′ represents the required force, and F′, which in this case acts upward or against the tendency to motion, the friction.

$$\Sigma X = X_W + X_{P'} + X_{F'} = 0.$$

$$-W \cos 60° - P' + F' = 0 \ldots\ldots\ldots\ldots\ldots\ldots \quad (6.)$$

$F' = N.\mu = 50\sqrt{3}$, which if substituted in (6.) gives

$$-100.\tfrac{1}{2} - P' + 50\sqrt{3} = 0.$$

$$P' = 50\sqrt{3} - 50.$$

A body weighing 50 pounds rests on a plane surface which is inclined to the horizontal at an angle of 45°. The coefficient of friction for body and plane is $1/\sqrt{3}$. What is the angle of repose? Find what force must be exerted on the body in a direction parallel to the plane to keep the body just on the point of sliding up.

The coefficient of friction = tangent of angle of repose.

$$= \frac{1}{\sqrt{3}} = \tan 30^\circ;$$

i.e., the angle of repose in this case is 30°.

It is evident that the body would, if left to itself, slip down the plane because the plane is inclined past the angle of repose.

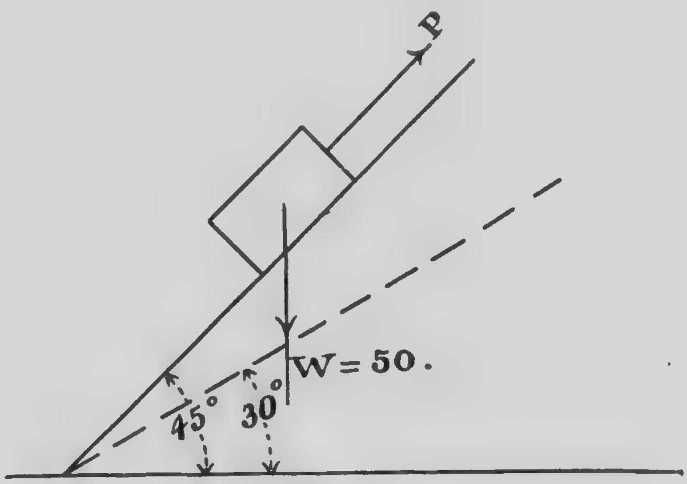


Fig. 56.

Fig. 56 represents the condition of affairs.

$$\Sigma X = X_W + X_P + X_F = 0.$$
$$-50 \cos 45^\circ + P - F = 0 \ldots\ldots\ldots\ldots\ldots\ldots (7.)$$

$$F = N.\mu.$$
$$= 50 \cos 45^\circ . \frac{1}{\sqrt{3}} = 50 . \frac{1}{\sqrt{2}} . \frac{1}{\sqrt{3}}.$$

Substituting this value of F into (7.),

$$-50\frac{1}{\sqrt{2}} + P - \frac{50}{\sqrt{2}}\cdot\frac{1}{\sqrt{3}} = 0.$$

$$P = \frac{50}{\sqrt{2}}\left(1 + \frac{1}{\sqrt{3}}\right).$$

CHAPTER XIII.

WIND PRESSURE.

Pressure on Surfaces Normal to Direction of Motion.

It will be primarily assumed that, for any stated velocity of wind, the motion of the air particles is uniform for all positions on any given cross section, no account being taken of interference of particles with one another or of eddies.

All calculations will be made at 60° Fahr. temperature and 760 cms. barometric pressure. Under these conditions one cubic foot of air weighs .078 pounds.

Let the velocity of the wind be "V" feet per second. Consider any cross section of one square foot area. The volume of air that will flow past this section in **one second** is evidently:—

$$V \times 1 \text{ cubic feet} \ldots\ldots\ldots\ldots \quad (1.)$$

or, in other words, since the flow is steady (uniform), V cubic feet of ai. flow past this section per second. Multiplying (1.) by .078 gives the flow in pounds per second as:—

$$V \times 1 \times .078 = .078\,V \text{ pounds per second..} \quad (2.)$$

Let dW be the weight of air flowing past this same section in a time "dT"; then, evidently,

$$dW = (.078\,V)\,dT.$$

Now, let attention be confined to an area of one square foot of any surface normal to the motion of the air particles. Considering the mass of air as being made up of a number of small masses of magnitude "dM," the action of each of these small particles as it strikes the normal surface is of an impulsive nature of duration "dT," such that:—

$$P.dT = dM.V. \ldots\ldots\ldots\ldots \quad (3.)$$

$$\text{but } dM = \frac{dW}{g}$$

$$= (.078\,V)\,dT \div 32.2.$$

Substituting this value of dM in (3.), it is seen that

$$P.dT = (.078\,V)\,dT.\,V/32.2\,;$$

or,

$$P = .078\,V^2/32.2. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.)$$

where P is the impulsive force acting on the normal surface; and, since the impulse is distributed over one square foot area, P may be said to be the resultant pressure per square foot.

In equation (4.) V is in feet per second, but it is usual to express wind velocities in miles per hour. Let V' be the wind velocity in miles per hour; then

$$V = V' \times 5280/60 \times 60.$$
$$= 1.47\,V'.$$

Substituting this value of V in (4.), we see that

$$P = .078\,(1.47\,V')^2/32.2.$$
$$= .0052\,V'^2,$$

which result, as would be reasonably expected, is higher than practical experiments show.

PROBLEMS IN APPLIED STATICS.

CHAPTER XIV.

MISCELLANEOUS PROBLEMS.

If the Vector Polygon be drawn for a set of forces which are in equilibrium, the polygon must close.

Let Fig. 58 represent a chain passed through a pulley to which is attached a hook, as shown. If the chain is inclined to the horizontal at 60° on each side of the block, what stress will be induced in the chain by hanging a weight of 3,464 pounds on the hook?

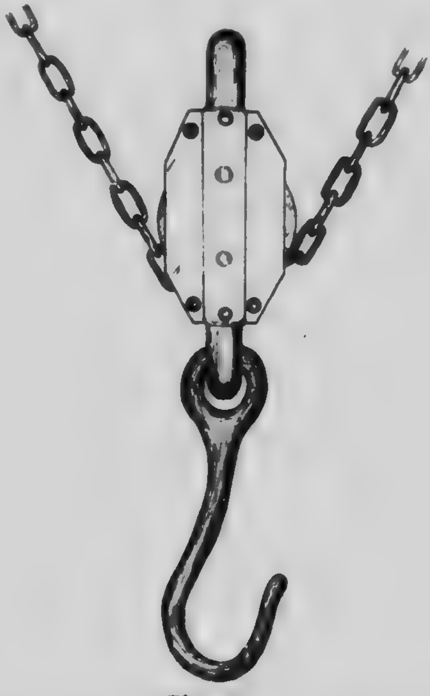


Fig. 58.

For all practical purposes the condition of affairs is such as indicated in the Statical Diagram (Fig. 59), a known force AB of 3,464 pounds acting vertically

downward, and two unknown forces, BC and CA, acting in directions which are inclined at 60° to the horizontal.

It is required to find the magnitudes of the two forces, BC and CA.

The three forces AB, BC, and CA are in equilibrium; therefore, if their vector polygon be constructed, it must close.

To construct the Vector Polygon:—

From any initial point A (Fig. 60) draw a line AB to represent completely the vertical force of 3,464 pounds, which is known as the force AB, A and B being

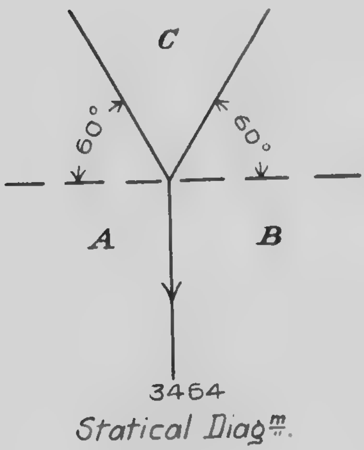


Fig. 59.

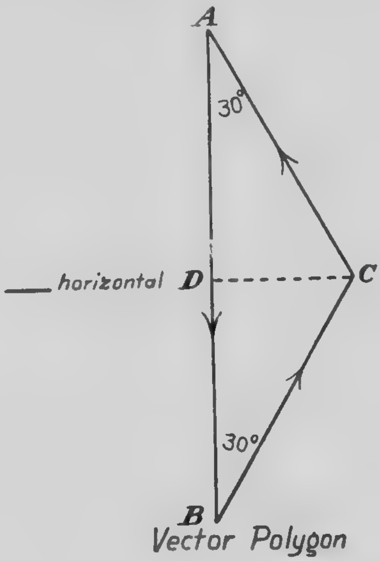


Fig. 60.

the letters on the areas to each side of the line representing the force in the Statical Diagram. (Such a system of lettering is known as Bow's Notation.) From B (Fig. 60) a line is drawn to represent the known direction of the so-called unknown force BC (Fig. 59).

Now, because the vector polygon must close, we know that the line representing the unknown force CA must pass through the initial point A (Fig. 60). Therefore, from A (Fig. 60) draw a line making 30° with AB; i.e., to represent the direction of the force CA. This line intersects the last line, drawn from B, at C.

BC and CA (Fig. 60) represent completely the hitherto unknown forces, and, since the sense marks in

a vector polygon must point continuously from the initial to the final point (which in this case are coincident), the senses of BC and CA are as indicated by the arrowheads.

If now, the sense marks as found in Fig. 60 are placed on the representations of BC and CA in the Statical Diagram, it is seen that both BC and CA act away from the point. The chain is, therefore, in tension (tends to tear apart). This last fact might easily have been seen in the first place, for a chain could not transmit other than tensile forces; but it is dangerous practice to assume without proof, conditions which seem to be self-evident.

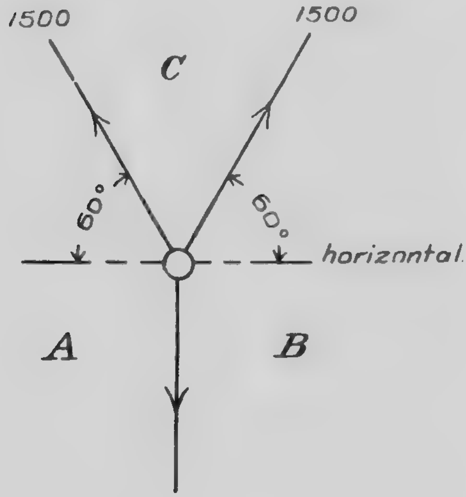


Fig. 61.

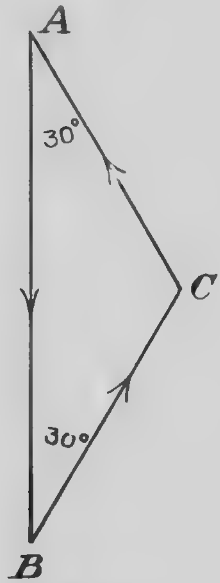


Fig. 62.

In order to find the magnitude of BC and CA, drop CD perpendicular on AB.

$$AD = DB.$$

Since AB represents 3,464 pounds,

$$AD = 1732 = DB.$$

$$BC = \frac{DB}{\cos 30^\circ} = \frac{1732}{1.732} \times 2 = 2000.$$

$$CA = \frac{AD}{\cos 30^\circ} = \frac{1732}{1.732} \times 2 = 2000.$$

If Fig. 60 has been constructed to scale, the lines BC and CA should each scale 2,000 pounds.

From the preceding it is seen, then, that the chain is **in tension** to the extent of 2,000 pounds.

The problem might have been stated in this manner.

If the chair (Fig. 58) can safely stand tension to the extent of 1,500 pounds, what is the maximum weight that may be placed on the hook?

Let the chain be stressed in tension to its limit of 1,500 pounds. Choosing a point where the line of action of the force due to the weight intersects the centre line of the chain, practically, the condition at this point is such as indicated by the statical diagram (Fig. 61), the chain exerting a tensile force of 1,500 pounds at each side of the point, and the weight, of as yet unknown magnitude, acting vertically downward. These three forces are in equilibrium; therefore, their vector polygon, if constructed, must close.

If BC and CA (Fig. 62) be drawn to represent accurately the two tensile forces exerted by the chain, it is

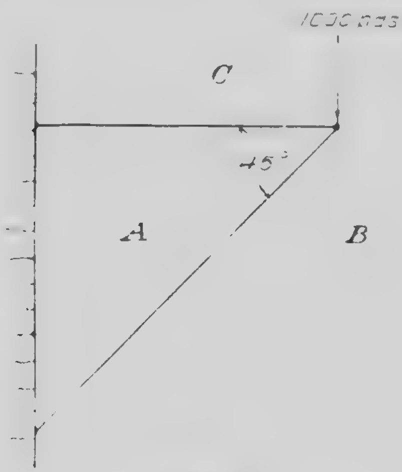


Fig. 63.

evident that the line joining A to B must represent the force exerted by the weight, for if BC and CA have been properly constructed, AB closes the diagram and represents a vertical force acting downward.

If a perpendicular be drawn from C (Fig. 62) to AB, it may be geometrically shown that—

$$AB = \frac{1{,}500}{2} \times \sqrt{3} + \frac{1{,}500}{2} \times \sqrt{3} = 1{,}500\sqrt{3} \text{ pounds.}$$

The maximum weight, then, that may be placed on the hook is $1{,}500\sqrt{3}$ pounds.

Let Fig. 63 represent diagrammatically a small cantilever bracket, the load which it supports being equivalent to 1,000 pounds applied at the outside joint as indicated. **It is required to find the stress in the members of the truss.**

Consider the forces acting at the point ABC of the frame. There is a known force BC of 1,000 pounds acting vertically downward, and two unknown forces,

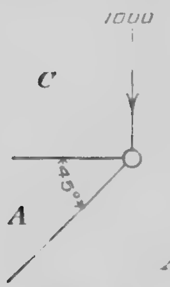


Fig. 64.

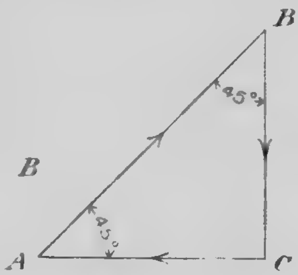


Fig. 65.

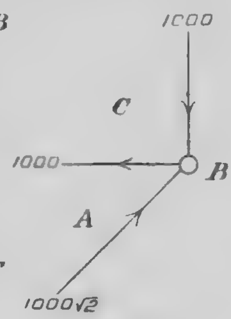


Fig. 66.

CA and AB, whose lines of action are known. The Statical Diagram (Fig. 64) represents this condition of affairs.

From any point B (Fig. 65) construct BC to represent the known vertical force BC.

Now, **since the forces under consideration are in equilibrium, their vector polygon must close.** Therefore, from C and B (Fig. 65) draw lines to represent the directions of the forces CA and AB, respectively. These lines intersect at A. Evidently, CA and AB (Fig. 65) represent the unknown forces CA and AB, the sense marks being continuous from the initial to the final point of the polygon as indicated.

The triangle BCA (Fig. 65) is, from construction, a $45°$ right angled triangle; therefore, since BC represents 1,000 pounds, CA and AB represent forces of 1,000 pounds and $1{,}000\sqrt{2}$ pounds, respectively.

By drawing a new statical diagram (Fig. 66) and placing on it all the above deduced quantities, it is seen that the member CA exerts a force away and the member AB a force against the point being considered.

The member CA is, therefore, in tension (resists being torn apart) to the extent of 1,000 pounds, and the member AB in compression (resists being compressed) to the extent of $1{,}000\sqrt{2}$ pounds.

If a set of forces is in equilibrium, the algebraic sum of the moments of the forces about any point is equal to zero.

Fig. 80 represents diagrammatically the method of arranging a system of levers in a weighing scale. Of course, there are certain constructional details which

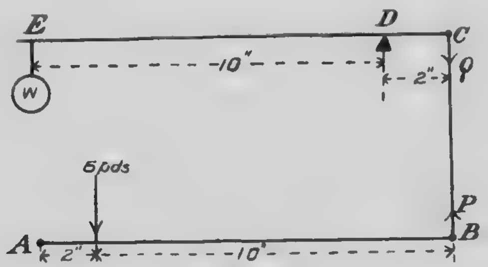


Fig. 80.

cannot be represented in such a figure, but nevertheless this problem illustrates the principle involved in all weighing scales and machines for testing the strength of materials.

AB and EC are two levers, the former being hinged at A and the latter resting on a knife edge D. The vertical rod BC is connected to these two levers at B and C by pin joints to allow of free motion of the system.

A force of six pounds is exerted on the lever AB at a distance of two inches from the hinge A. What is the magnitude of the weight W placed on the upper lever at E such that the whole system is in equilibrium?

Consider the lever AB. It is acted upon by three forces; the vertical force of six pounds, an unknown

vertical force P at B, and the reaction at the hinge A. These forces are in equilibrium; therefore, if moments be taken about any point, $\Sigma M = 0$.

Take moments about the point A.

$$\Sigma M = (\text{reaction at } A \times \text{zero}) + 6 \times 2 + P \times 12 = 0 \ldots\ldots\ldots (1.)$$

Since we do not know whether P acts upward or downward, the moment of P is assumed positive.

$$P \times 12 = -12.$$
$$P = -1.$$

This negative result shows that the moment of P about A is negative; i.e., P acts upward at B as indicated by the arrowhead, and is of magnitude one pound.

The rod BC exerts a force away from B. It must, therefore, exert an equal and opposite force Q at its other extremity C; i.e., at C there is a vertical force of one pound acting downward as indicated by the arrowhead.

Consider the lever EC. It is acted upon by three forces in equilibrium, viz., the force W acting at E, the reaction of the knife edge at D, and the known vertical force acting at C. For this set of forces, ΣM about any point equals zero.

Take moments about D:—

$$\Sigma M = Q \times 2 + (\text{reaction at } D \times \text{zero}) + W \times 10 = 0 \ldots\ldots (2.)$$

Putting in the known value of $Q = 1$, we get:—

$$W \times 10 = -2.$$
$$W = -1/5.$$

That is, W is a weight of 1/5 pound, and acts in such a manner as to give a negative moment about D. W, therefore, acts downward.

The reader may question why the moments of P and W in equations (1) and (2) were written positive when his experience would most likely say that both these quantities should be negative in compliance with the convention of positive and negative moments. It must be kept in mind, however, that although experience says that P must act upward and W downward, thereby giving negative moments, it is dangerous practice to assume these self-evident facts in the first place. It is generally safer to assume these forces totally unknown and then by the reasoning of the problem deduce how they act. The moments of unknown forces are, of

course, assumed positive, just as an unknown quantity in an ordinary algebraic equation is assumed positive.

Fig. 81 represents an ordinary bell crank.

If a force P of 100 pounds be exerted at the joint of the upper arm as indicated, what force S must be exerted at the joint of the other arm in order to preserve equilibrium?

Consider the crank as the body acted upon. The forces acting on this body are P, S, and the reaction Q

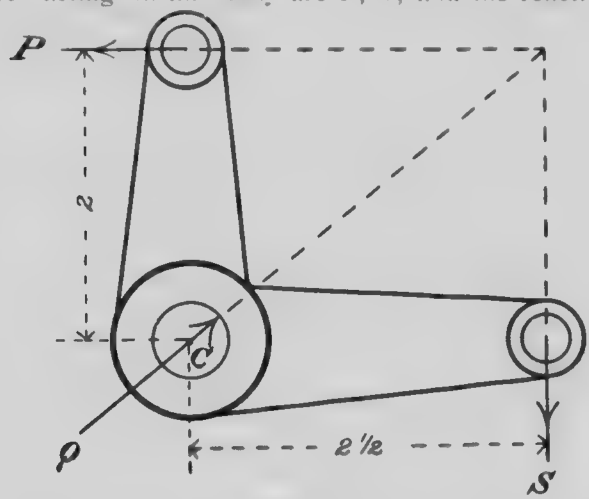


Fig. 81.

of the pin on the crank, and for this problem P and S will be considered as acting perpendicularly to the arms of the crank. Since these forces are in equilibrium, their lines of action must intersect at a common point as indicated. (Three forces in equilibrium must act at a point.) These forces being in equilibrium, $\Sigma M = 0$.

Take moments about the point C, the centre of rotation of the crank.

$$\Sigma M = M_P + M_Q + M_S = 0.$$

$$-P.\ 2 + Q.\ 0 + S.\ 2\tfrac{1}{2} = 0.$$

Putting in the value of $P = 100$, we get:

$$-100 \times 2 + 0 + 2\tfrac{1}{2}.\ S = 0.$$

$$S = 80.$$

The positive result shows that the M_S about C is positive; i.e., S acts as indicted on the diagram.

Now, although S is the force which must act with P and Q to give equilibrium, it must be clearly under-

stood that if a force P be exerted at one end of the crank as shown, that **the crank** will exert at the other end a force **equal but opposite to S** on any body to which it may be fastened. The body in resisting this will exert the force S as shown.

To Find the Reaction Q:—

Apply either the equation $\Sigma X = 0$ or $\Sigma Y = 0$ to the set of forces P, Q, and S. The value of the sine or cosine of the angle of inclination of Q may be found from the given distances of P and S from C, these distances forming the sides of a right angled triangle, one angle of which is the required inclination. If the hypotenuse of this triangle be calculated, the required sine or cosine may be obtained.

CHAPTER XV.

SIMULTANEOUS USE OF EQUATIONS.
$\Sigma X = 0$ and $\Sigma Y = 0$.

In the work previous to this, it has always been possible to determine the unknown forces of a set, one at a time, by the independent use of either the equations $\Sigma X = 0$ or $\Sigma Y = 0$. In the case that will be taken up next, and in many other cases, it is only possible to arrive at results by the simultaneous use of the two equations $\Sigma X = 0$ and $\Sigma Y = 0$, and in such cases certain precautions must be observed.

The roof truss (Fig. 96) is known as a German Truss. This form of structure is obsolete as far as prac-

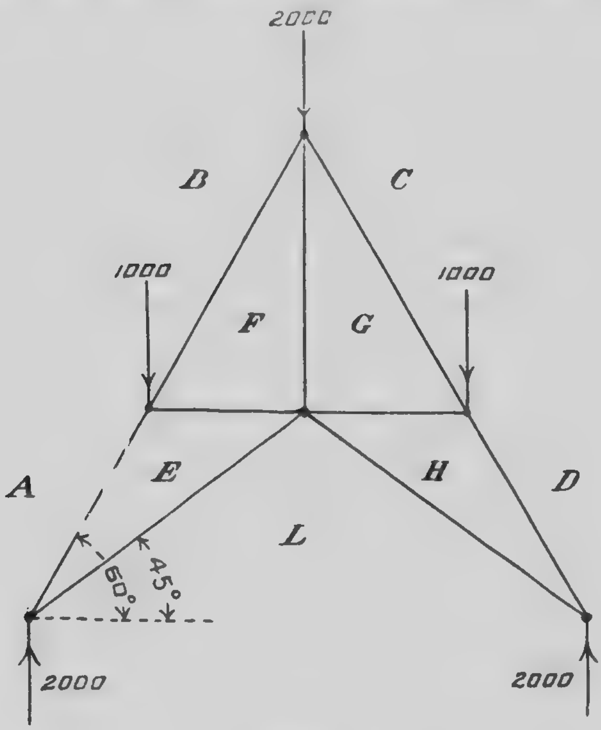


Fig. 96.

tice is concerned, but is taken up because it serves very well the purpose of illustrating the point under discussion.

Considering the forces acting **on** the truss and applying the equation $\Sigma M = 0$, taking moments about a point in the line of action of either abutment reaction, it is seen that the abutment reactions are both equal to 2,000 pounds.

Consider the forces acting at the point LHD (Statical Diagram, Fig. 97). Since these forces are in equilibrium, $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma M = 0$.

$$\Sigma X = X_{LH} + X_{HD} + X_{DL} = 0.$$

LH and HD being both unknown, assume their X's positive.

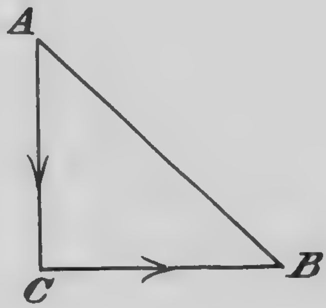


Fig. 98.

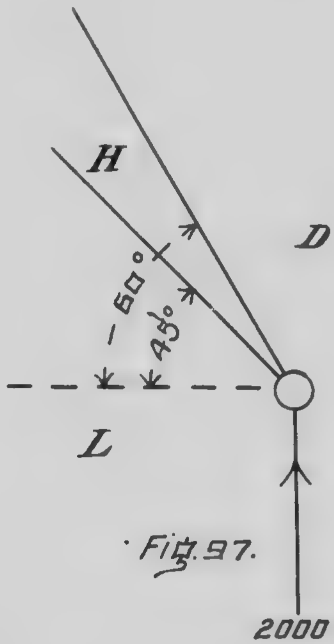


Fig. 97.

$$LH \cos 45^\circ + HD \cos 60^\circ + 0 = 0.$$

$$LH.\frac{1}{\sqrt{2}} + HD.\frac{1}{2} = 0.$$

$$LH = -\frac{\sqrt{2}}{1}.\frac{HD}{2.}$$

$$= -\frac{HD}{\sqrt{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1.)$$

It is evident that in order to solve for the two unknowns, LH and HD, another equation must be formed in terms of the same unknowns, this equation to be used simultaneously with equation (1.). Using $\Sigma Y = 0$ as the other equation:—

$$\Sigma Y = Y_{LH} + Y_{HD} + Y_{DL} = 0.$$

Since the equations ΣX and ΣY are being used simultaneously, we are not at liberty to assume the Y's of the unknown forces LH and HD positive (as would ordinarily be done) without first considering the assumptions in the first equation. **The assumptions as to the signs of the Y's of the unknown forces must be in accordance with the assumptions as to the signs of the X's of the unknown forces in the first place.** Referring to Fig. 98, let AB represent the line of action of the force LH (Fig. 97), and, since the X_{LH} was assumed positive, the line CB with sense mark to the right evidently represents the assumed X_{LH}. Now, although the magnitude of X_{LH} is unknown, since the magnitude of the force LH is unknown, yet it is seen from Fig. 98 that, irrespective of the magnitude of X_{LH}, if the X_{LH} is assumed positive, the Y_{LH} must be assumed negative as represented by such a line as AC (Fig. 98). (It is seen from Fig. 98 that no matter what the length of the line representing X_{LH} be, the line representing the corresponding Y_{LH} will have its sense mark pointing downward in order to complete the diagram.)

In the same way, it is evident that Y_{HD} must be assumed negative if X_{HD} be primarily assumed positive.
$\Sigma Y = -LH \sin 45° - HD \sin 60° + 2{,}000 = 0.$

$$-LH \frac{1}{\sqrt{2}} - HD \frac{\sqrt{3}}{2} + 2{,}000 = 0 \ldots\ldots (2.)$$

Substitute value of LH from (1.) into (2.).

$$-\left(-\frac{HD}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{2}} - HD \cdot \frac{\sqrt{3}}{2} = -2{,}000.$$

$$\frac{HD}{2} - \frac{HD}{2} \cdot \sqrt{3} = -2{,}000.$$

$$\frac{HD}{2}(1 - \sqrt{3}) = -2{,}000.$$

Multiplying both sides of the equation by — 1 :—

$$\frac{HD}{2}(\sqrt{3} - 1) = 2.000.$$

$$HD = \frac{4,000}{(\sqrt{3} - 1)} \quad \ldots\ldots\ldots\ldots\ldots \quad (3.)$$

From the positive sign of the result, it is seen that the assumptions as to the X_{HD} and Y_{HD} are correct. **The Y_{HD} was assumed negative and X_{HD} positive,** which is seen to be correct; the force HD, therefore, acts against the point; i.e., the member HD is in compression $\frac{4,000}{(\sqrt{3} - 1)}$ pounds.

Substituting the value of HD from (3.) into (1.):—

$$LH = -\frac{HD}{\sqrt{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.)$$

By substitution:—

$$LH = -\frac{4,000}{(\sqrt{3} - 1)\sqrt{2}}$$

It is evident [illegible] the negative sign of the result that the assumed s[illegible]s of X_{LH} and Y_{LH} are wrong; i.e., X_{LH} is negative and Y_{LH} positive. The force LH, therefore, acts away from the point; i.e., the member LH is in tension $\frac{4,000}{(\sqrt{3} - 1)\sqrt{2}}$ pound

The same results would have been obtained had the equation $\Sigma Y = 0$ been used first, although the work would have been slightly more involved. The point of the preceding discussion is that, **if the equations have to be used simultaneously, the assumptions as to signs in the second equation must be in accordance with the given lines of action of the unknown forces and the assumed signs in the first equation.**

The construction and loading of the truss (Fig. 96) are symmetrical about the same axis. From this fact we are at liberty to say that the left-hand half of the truss will have the same stress in any of its members as the corresponding members in the other half of the truss. It would, therefore, be unnecessary to consider the forces

at the point LEA in order to find the stress in the members LE and EA (LE will be in tension and EA in compression to the same extent as the members LH and HD, respectively). It is advisable, however, that the reader go through the analytical determination of these stresses.

Consider the forces acting at the point LEA.

$$\Sigma X = X_{LE} + X_{EA} + X_{AL} = 0.$$

$$LE \cos 45^\circ + EA \cos 60^\circ + 0 = 0.$$

$$LE = -\frac{EA}{\sqrt{2}} \ldots \quad (4.)$$

$$\Sigma Y = Y_{LE} + Y_{EA} + Y_{AL} = 0.$$

In accordance with the assumptions as to the X_{LE} and X_{EA}, the Y_{LE} and Y_{EA} will both be positive.

$$\Sigma Y = LE \sin 45^\circ + EA \sin 60^\circ + 2{,}000 = 0 \ldots \quad (5.)$$

substituting value of LE from (4.) into (5.).

$$\left(-\frac{EA}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{2}} + EA\frac{\sqrt{3}}{2} + 2{,}000 = 0.$$

$$EA\frac{(\sqrt{3}-1)}{2} = -2{,}000.$$

$$EA = -\frac{4{,}000}{(\sqrt{3}-1)} \quad (6.)$$

From the negative result, the assumptions as to X_{EA} and Y_{EA} are seen to be wrong. X_{EA} and Y_{EA} are, therefore, both negative; i.e., EA acts against the point. The member EA is in compression $\frac{4{,}000}{(\sqrt{3}-1)}$ pounds.

Substituting the value of EA from (6.) into (4.):—

$$LE = -\left(-\frac{4{,}000}{(\sqrt{3}-1)}\right) \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{4{,}000}{(\sqrt{3}-1)} \cdot \frac{1}{\sqrt{2}}$$

It follows from the positive result that the assumptions as to X_{LE} and Y_{LE} are correct; i.e., X_{LE} and Y_{LE} are positive. LE, therefore, acts away from the point. The member LE is in tension $\frac{4{,}000}{(\sqrt{3}-1)} \cdot \frac{1}{\sqrt{2}}$ pounds.

CHAPTER XVI.

DETERMINATION OF STRESS IN BRIDGE AND ROOF TRUSS MEMBERS.

In all the following problems on the determination of the stress in the members of a framed structure, **the joints of the truss must be considered as frictionless circular pin joints, perfectly fitted so as to allow of no slack motion,** these being the assumptions upon which the theory is based. So, then, whenever the reader encounters the statement to consider the forces acting at a point in a truss, he must bear in mind that he is really considering **the forces which act on the pin of the joint,** for in any statical problem, when a set of forces is being considered, there must always be some body acted upon. In these cases the pin is that body, and the members are the bodies acting on it. It may be pointed out, however, **that because of this construction.** the lines of action of the forces acting on the pin intersect at the centre of the pin, and it is this fact that is referred to in the statement: "To consider the forces acting at a point." With the exception of the case of three forces in equilibrium, which, of course, **must** have directions intersecting at a common point, it must not be thought that the forces acting at a joint in a truss must act through the centre of the pin in order to be in equilibrium. **They merely do so,** as pointed out before, **because of the construction of the joint.**

Find the stress in the members of a cantilever such as indicated in Fig. 67, the panels of which are equilateral triangles.

Consider the point BCA. The forces acting at this point are in equilibrium, the conditions being such as indicated in the Statical Diagram (Fig. 68). If the Vector Polygon for this set of forces be constructed, it must close.

The reader must bear in mind the **important** fact that these Statical Diagrams merely represent the conditions **at the point being considered.** The lines indicating the directions of the known and unknown forces are **the lines of action of forces acting on the pin.**

In the following discussion it will be assumed that the relative lines of action of the forces being considered are accurately represented in the Statical Diagram; so that in constructing the Vector Polygon for a set of

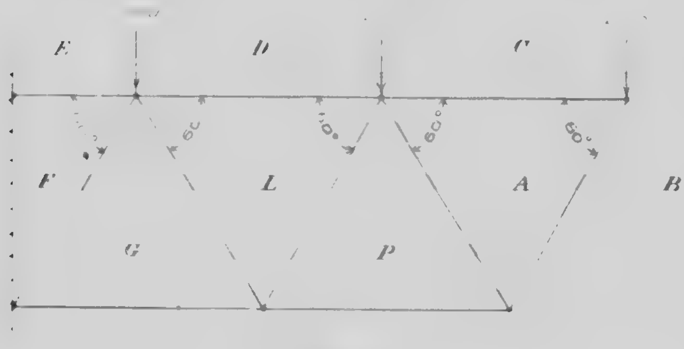
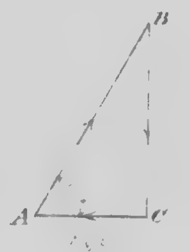
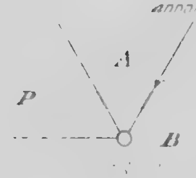


Fig. 67.

forces, all that will be necessary, in order to represent the direction of any force, will be to draw a line parallel to the line representing the force in the Statical Diagram.

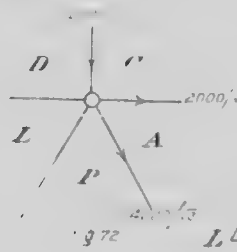


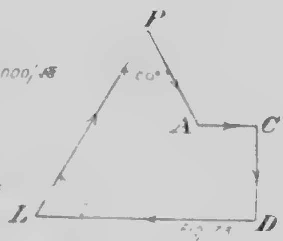


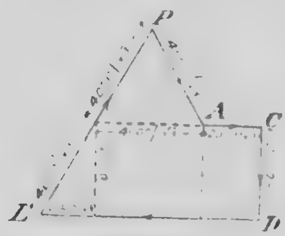
Construct BC (Fig. 69) to represent the known vertical force. From C and B draw lines parallel to CA and AB (Fig. 68), respectively, to represent the direc-

tions of the unknown forces CA and AB. These lines intersect at A. Evidently CA and AB (Fig. 69) represent the unknown forces CA and AB, their magnitudes being $2{,}000/\sqrt{3}$ pounds and $4{,}000/\sqrt{3}$ pounds, respectively.

If the sense marks of CA and AB found in Fig. 69 be placed on the Statical Diagram (Fig. 68), it is seen that the member CA exerts a force away and the member AB a force against the pin. The member CA is, therefore, in tension, and the member AB in compression, to the extent of $2{,}000/\sqrt{3}$ pounds and $4{,}000/\sqrt{3}$ pounds, respectively.







Consider the point BAP. Since the member BA is in compression, it must exert a force against the point BAP. This, together with the other conditions at the point, is represented in the Statical Diagram (Fig. 70).

BA (Fig. 71) represents the known force BA. From A and B draw lines parallel to the directions of AP and PB (Fig. 70), respectively. These lines intersect at P. Then AP and PB (Fig. 71), with sense marks running continuously as indicated, must represent the hitherto unknown forces AP and PB.

If the sense marks of AP and PB as found in Fig. 71 be placed on the corresponding lines of action

8

of AP and PB in Fig. 70, it is seen that AP acts away from the point and PB against it.

The member AP is, therefore, in tension $4{,}000/\sqrt{3}$ pounds, and the member PB in compression $4{,}000/\sqrt{3}$ pounds.

Consider the point PACDL. Acting at this point are three known forces (the load and two forces exerted by the tension members PA and AC) and two unknown forces as indicated in the Statical Diagram (Fig. 72).

PA, AC, and CD (Fig. 73) represent the known forces PA, AC, and CD. From D and P, respectively, draw lines parallel to the directions of DL and LP (Fig. 72), intersecting at L. DL and LP (Fig. 73) represent, respectively, the unknown forces DL and LP, acting as indicated by the continuously pointed sense marks. In order to avoid confusion, another diagram (Fig. 74) has been constructed, showing how to arrive at the magnitudes of DL and LP. From this diagram it is seen that DL and LP are forces of $8{,}000/\sqrt{3}$ pounds each.

The force DL evidently acts away from the point and LP against the point. The member DL is, therefore, in tension $8{,}000 \div \sqrt{3}$ pounds, and the member LP in compression $8{,}000 \div \sqrt{3}$ pounds.

Find the stress in the remaining members LG, GB, EF, and FG.

These stresses may be found by first considering the point BPLG, and then the point GLDEF.

Fig. 75 represents the condition at the point BPLG. In drawing the Vector Polygon for this point it is found that the line GB (Fig. 76) coincides for part of its length with the line BP already drawn. In order to help the reader, the portion of GB which would coincide with BP has been shown by a dotted line. The diagram reads: BP, PL, LG, and GB. From this it is seen that the member LG is in tension $8{,}000/\sqrt{3}$ pounds and the member GB in compression $12{,}000/\sqrt{3}$ pounds.

Figs. 77 and 78 are, respectively, the Statical Diagram and Vector Polygon for the point GLDEF. The member EF is in tension $17{,}000/\sqrt{3}$ pounds and the member FG in compression $10{,}000\sqrt{}/3$ pounds.

To Construct a Stress Diagram.

In the last problem a new Vector Polygon was constructed for each set of forces being considered. If these diagrams are drawn to a suitable working scale, they spread over a considerable area, and also, in transferring magnitudes to a new line, there is the liability of an error which may or may not be cumulative in its effect throughout the problem. It is possible, however, to condense these diagrams and at the same time restrict the liability to error in magnitudes by drawing what the writer chooses to call a **Continuous Vector Diagram**, or, as it is often referred to, a **Stress Diagram.**

It may be pointed out that it will help to simplify the problem if the reader will construct, on a loose sheet of paper, the Vector Polygons (Figs. 69, 71, 73, 76, and

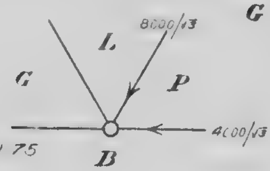


Fig 75

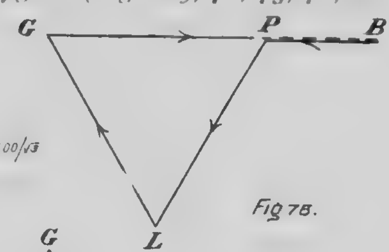


Fig 76.

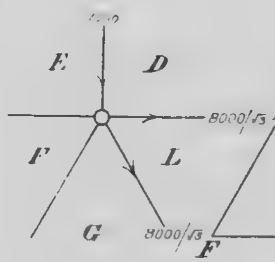


Fig 77.

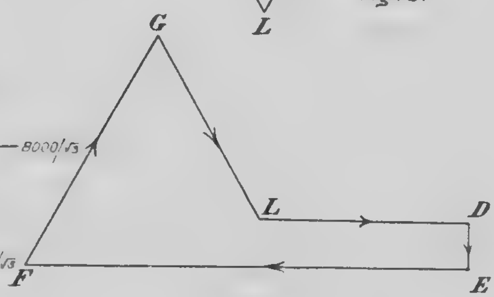


Fig 78

78), using the same scale of forces in each case. These polygons may then be referred to in drawing the following diagram. The reader must bear in mind that **the same scale of forces will be used throughout the following problem:—**

Considering the Statical Diagram (Fig. 68), it is seen that the triangle BCA (Fig. 79) may be taken as a Vector Polygon for the forces acting at the point BCA of the truss (Fig. 67). Instead of placing sense marks on this polygon to indicate how the forces act, a thin line is used to represent that the force CA acts away from the point, a thick line to indicate that the force AB acts

against the point, and a double thin line to denote that the force BC is due to a load. This system of representing forces will be adhered to throughout the problem.

Taking next the point BAP, it is seen from the Statical Diagram (Fig. 70) that the force BA acts against the point. Referring to Fig. 79, it is found that there

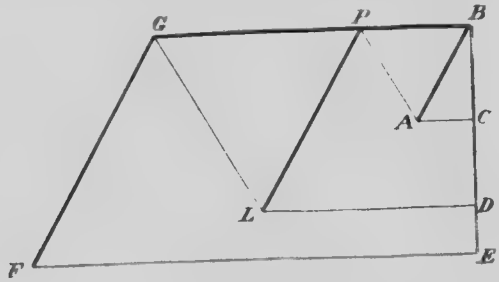


Fig. 79.

is already drawn a line which may represent this force, provided the letters at the extremities of this line be read the proper way to correspond with the sense of the force as shown on the Statical Diagram namely, the thick line BA. The lines AP and PB complete the Vector Polygon for this point. AP, acting away from the pin, is represented by a thin line, and PB, acting against the pin, by a thick line.

Acting at the point PACDL are three known forces, PA, AC, and CD, as shown in the Statical Diagram (Fig. 72). It is seen that the thin lines PA and AC (Fig. 79) are so constructed that they may represent the forces PA and AC. From C (Fig. 79) is drawn a double line CD to represent the force CD due to the load. The lines DL and LP represent, respectively, the unknown forces DL and LP. If sense marks were placed on the diagram, DL and LP would be found to be acting away and against the point, respectively, as indicated by the thin and thick lines. The diagram for this point reads PA, AC, CD, DL, and LP.

From the Statical Diagram for the point BPLG (Fig. 75) it is seen that the two forces BP and PL are due to compression members. They may, therefore, be represented by the thick lines BP and PL (Fig. 79), since these lines are properly constructed to represent their magnitudes and directions. The Vector Polygon for the point is completed by the lines LG and GB, the diagram reading BP, PL, LG, and GB.

The reader is advised to construct the remainder of the diagram for the point GLDEF for himself. The diagram when drawn should read, GL, LD, DE, EF, and FG.

Doubtless, the reader will see that the construction of this Stress Diagram is merely a process of piecing together the Vector Polygons for the various points of the truss.

It should be noticed that the lines of the Stress Diagram, since they have no sense marks placed on them, really represent the magnitude of the stress in the various members; e.g., the line LG represents the stress in the member LG.

In the two preceding problems the determination of the stress in the various members of the cantilever truss (Fig. 67) was arrived at by graphical means, although a little trigonometry was occasionally made use of. It is possible, however, to arrive at these quantities analytically as indicated in the following discussion. In order to simplify matters, the reader is advised to draw the Statical Diagrams (Figs. 68, 70, 72, 75, and 77) on a loose sheet of paper so that they may be readily referred to.

The following facts must be kept in mind throughout the problem:—

The **magnitude** of the X of any force = (magnitude of force) × (cosine of angle of inclination of force to the horizontal).

The **magnitude** of the Y of any force = (magnitude of force) × (sine of angle of inclination of force to the horizontal).

Also, if a set of forces be in equilibrium, $\Sigma \mathbf{X} = \mathbf{0}$, **and** $\Sigma \mathbf{Y} = \mathbf{0}$.

Analytical Determination of the Stress in the Members of the Cantilever (Fig. 67).

Consider the forces acting at the point BCA as shown in the Statical Diagram (Fig. 68). These forces are in equilibrium. Therefore, $\Sigma X = 0$; $\Sigma Y = 0$.

$$\Sigma Y = Y_{BC} + Y_{CA} + Y_{AB} = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (1.)$$

Now, since CA and AB are unknown forces, the Y_{CA} and Y_{AB} will be assumed positive.

$$Y_{BC} = -BC \sin 90^\circ = -BC = -2{,}000.$$
$$Y_{CA} = CA \sin 0^\circ = 0.$$
$$Y_{AB} = AB \sin 60^\circ = AB.\frac{\sqrt{3}}{2} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.)$$

Substituting these values in equation (1.).

$$\Sigma Y = -2{,}000 + 0 + AB.\frac{\sqrt{3}}{2} = 0.$$

$$AB = 2{,}000.\frac{2}{\sqrt{3}} = \frac{4{,}000}{\sqrt{3}}$$

If, now, this true value of AB, which is a positive quantity, be substituted for AB in equation (2.), it is seen that the assumed positive sign of the Y of AB remains unchanged. The assumed sign of the Y of AB is, therefore, correct; i.e., the Y of AB is positive.

If Y_{AB} is positive, i.e., acts upward, AB is found to be a force acting against the pin. The member AB is, therefore, in compression $\frac{4{,}000}{\sqrt{3}}$ pounds.

For the same point:—

$$\Sigma X = X_{BC} + X_{CA} + X_{AB} = 0 \quad \ldots\ldots\ldots\ldots \quad (3.)$$
$$X_{BC} = BC \cos 90^\circ = 0.$$
$X_{CA} = CA \cos 0^\circ = CA.$ X_{CA} assumed positive since CA is unknown............ (4.)

$$X_{AB} = AB \cos 60^\circ = AB.\tfrac{1}{2}. = \frac{4{,}000}{\sqrt{3}}.\frac{1}{2} \quad \frac{2{,}000}{\sqrt{3}}$$

Substituting these values in (3.).

$$\Sigma X \quad 0 + CA + \frac{2{,}000}{\sqrt{3}} = 0.$$

$$CA = -\frac{2{,}000}{\sqrt{3}}.$$

Substituting this true value of CA (a negative quantity) into equation (4.), the X_{CA} is seen to be negative. If X_{CA} is negative, CA must be a force acting away from the pin. From this, then, the member CA is seen to be in tension $2{,}000 \sqrt{3}$ pounds.

The forces acting at the point BAP, as shown in the Statical Diagram (Fig. 70), are in equilibrium.

$$\Sigma Y = Y_{BA} + Y_{AP} + Y_{PB} = 0.$$

$$-BA \sin 60^\circ + AP \sin 60^\circ + PB \sin 0^\circ = 0.$$

$$-\frac{4{,}000}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} + AP\frac{\sqrt{3}}{2} + 0 = 0.$$

$$AP = \frac{4{,}000}{}.$$

From the positive sign of this result the Y_{AP} is seen to be positive; i.e., the force AP acts away from the pin. The member AP is, therefore, in tension $\frac{4{,}000}{\sqrt{3}}$ pounds.

For the same point:—

$$\Sigma X = X_{BA} + X_{AP} + X_{PB} = 0.$$

$$-BA \cos 60^\circ - PB \cos 60^\circ + PB \cos 0^\circ = 0.$$

$$-\frac{4{,}000}{\sqrt{3}} \cdot \frac{1}{2} - \frac{4{,}000}{\sqrt{3}} \cdot \frac{1}{2} + PB = 0.$$

$$PB = \frac{4{,}000}{\sqrt{3}}.$$

The positive sign of the result shows that the X_{PB} is positive. PB must, therefore, act against the pin, placing the member PB in compression $\frac{4{,}000}{\sqrt{3}}$ pounds.

The forces acting at the point PACDL are in equilibrium, and act as represented in the Statical Diagram (Fig. 72).

$\sqrt{3}$

$$\Sigma Y = Y_{PA} + Y_{AC} + Y_{CD} + Y_{DL} + Y_{LP} = 0.$$

$$-PA \sin 60^\circ + AC \sin 0^\circ - CD \sin 90^\circ + DL \sin 0^\circ + LP \sin 60^\circ = 0.$$

$$-\frac{4{,}000}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} + 0 - 2{,}000 + 0 + LP \cdot \frac{\sqrt{3}}{2} = 0.$$

$$LP = \frac{8{,}000}{\sqrt{3}}.$$

The positive sign of the result shows that the Y_{LP} is positive; i.e., acts upward. In accordance with this, LP must act against the pin, placing the member LP in compression $8{,}000/\sqrt{3}$ pounds.

For the same point:—

$$\Sigma X = X_{PA} + X_{AC} + X_{CD} + X_{DL} + X_{LP} = 0.$$

$$PA \cos 60^\circ + AC \cos 0^\circ + CD \cos 90^\circ + DL \cos 0^\circ + LP \cos 60^\circ = 0.$$

$$\frac{4{,}000}{\sqrt{3}} \cdot \frac{1}{2} + \frac{2{,}000}{\sqrt{3}} + 0 + DL + \frac{8{,}000}{\sqrt{3}} \cdot \frac{1}{2} = 0.$$

$$DL = -\frac{8{,}000}{\sqrt{3}}.$$

The negative sign of the result shows that the X_{DL} (since the X equation is being used) is negative. DL, therefore, acts away from the pin; i.e., the member DL is in tension $\frac{8{,}000}{\sqrt{3}}$ pounds.

The reader is advised to try and determine the stresses in the remaining members before referring to the solution as herein given.

For the forces acting at the point BPLG (Statical Diagram, Fig. 75):—

$$\Sigma Y = Y_{BP} + Y_{PL} + Y_{LG} + Y_{GB} = 0.$$

$$0 - \frac{8{,}000}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} + LG \cdot \frac{\sqrt{3}}{2} + 0 = 0.$$

$$LG = \frac{8{,}000}{\sqrt{3}}.$$

From this positive result it is seen that Y_{LG} is positive. LG, therefore, acts away from the point; i.e., the member LG is in tension $\frac{8{,}000}{\sqrt{3}}$ pounds.

For the same point:—

$$\Sigma X = X_{BP} + X_{PL} + X_{LG} + X_{GB} = 0.$$

$$-\frac{4{,}000}{\sqrt{3}} - \frac{8{,}000}{\sqrt{3}} \cdot \frac{1}{2} - \frac{8{,}000}{\sqrt{3}} \cdot \frac{1}{2} + GB = 0.$$

$$GB = \frac{12{,}000}{\sqrt{3}}.$$

The X_{GB} is evidently positive. GB, it follows, acts against the pin. The member GB is, therefore, in compression $\frac{12,000}{\sqrt{3}}$ pounds.

Considering the point GLDEF (Statical Diagram, Fig. 77):—

$$\Sigma Y = Y_{GL} + Y_{LD} + Y_{DE} + Y_{EF} + Y_{FG} = 0.$$

$$-\frac{8,000}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} + 0 - 1,000 + 0 + FG \cdot \frac{\sqrt{3}}{2} = 0.$$

$$FG = \frac{10,000}{\sqrt{3}}.$$

The Y_{FG} being positive (from the positive result), FG must act against the pin; i.e., the member FG is in compression $\frac{10,000}{\sqrt{3}}$ pounds.

For the same point:—

$$\Sigma X = X_{GL} + X_{LD} + X_{DE} + X_{EF} + X_{FG} = 0.$$

$$\frac{8,000}{\sqrt{3}} \cdot \frac{1}{2} + \frac{8,000}{\sqrt{3}} + 0 + EF + \frac{10,000}{\sqrt{3}} \cdot \frac{1}{2} = 0.$$

$$EF = -\frac{17,000}{\sqrt{3}}.$$

The negative result shows that X_{EF} is negative. In accordance with this, EF acts away from the point. The member EF is, therefore, in tension $\frac{17,000}{\sqrt{3}}$ pounds.

It is seen that these results check those found by graphical means. It must be recognized, however, that the so-called graphical solutions taken up so far in these pages have not been entirely graphical. Instead of constructing the diagrams and finding the magnitudes of the forces by scaling the lengths of the lines composing the Vector Polygon, trigonometrical means have been used to get at these quantities. The solutions have, therefore, been partly analytical. The point of the discussion is that had purely graphical means been employed there would have been found an error in the magnitudes of the forces as compared to the values obtained

analytically, but this error is so small that in practice it does not amount to anything. Analytical results are, of course, perfectly accurate.

King Post Truss.

Fig. 82 represents a simple form of roof or bridge truss, known as a King Post Truss. It must be clearly understood, however, that the ordinary forms of roof bracing, consisting of the rafters and scantling tie-rods, which resemble the above truss in outline, do not really present the same problem as will be herein discussed. Usually, the rafters have the roof sheeting, shingles, etc., lying directly on them over their entire length. In this case, these rafters are inclined beams supporting a distributed load, and the stress in them will not be simple Compression, but a combination of both Tension and Compression due to bending. If, however, stringers or purlins be laid from truss to truss on the roofing system so as to lie at the joints of the trusses, or nearly so, and if the rafters and roofing be built on these stringers, the load will then be transferred to the truss merely at the joints, and the case may then be worked out by the methods taken up so far. Fig. 82 A

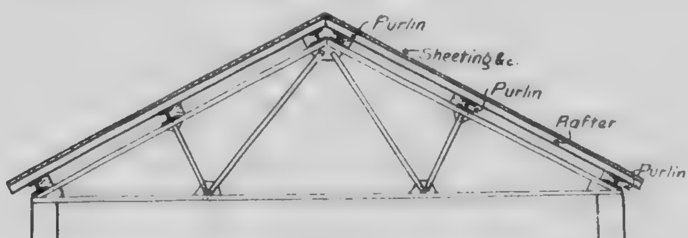


Fig 82 A.

illustrates, for another form of roof truss, the method of laying down purlins and building on them.

There is also a form of bridge truss very commonly met with on country roads which is sometimes called a King Post Truss. The structure referred to has a heavy timber beam laid between the abutments, and the trussing above merely serves to stiffen this beam. In this case, the beam, which seemingly corresponds to the horizontal tie-rod of the ordinary King Post Truss, is not in simple Tension, but has both Tension and Compression existing in it, and these stresses cannot be found by elementary methods.

Required to find the stress in the members of the truss (Fig. 82).

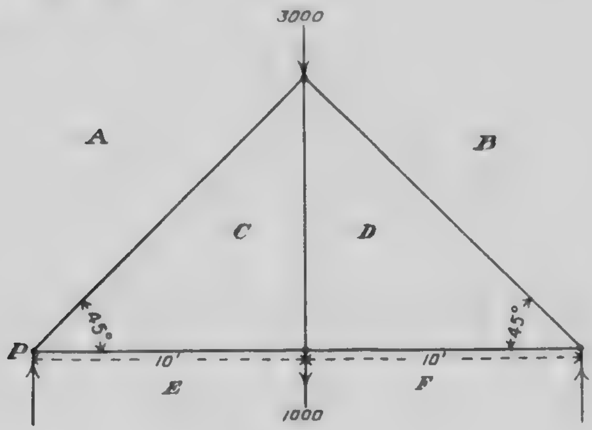


Fig. 82.

Analytical Solution :—

The whole truss is a body acted upon by a set of outside forces. These forces, the two loads AB and EF, and the two abutment reactions BF and EA, are in equilibrium; therefore, $\Sigma X = 0$; $\Sigma Y = 0$; $\Sigma M = 0$.

Take moments about a point in the line of action of one of the abutment reactions, say, the point P.

$$\Sigma M = M_{AB} + M_{EF} + M_{BF} + M_{EA} = 0.$$

Since BF is unknown, assume M_{BF} positive.

$$3{,}000 \times 10 + 1{,}000 \times 10 + BF \times 20 + EA \times 0 = 0.$$

$$BF = -2{,}000.$$

From the negative sign of the result, it is seen that M_{BF} about P is negative; i.e., BF acts upward.

$$\Sigma Y = Y_{AB} + Y_{EF} + Y_{BF} + Y_{EA} = 0.$$

$$-3{,}000 - 1{,}000 + 2{,}000 + EA = 0.$$

$$EA = 2{,}000.$$

From the positive result we infer that Y_{EA} is positive; i.e., EA acts upward.

Consider the forces acting on the pin at the point EAC. The known and unknown forces are represented in the Statical Diagram (Fig. 83).

$$\Sigma Y = Y_{EA} + Y_{AC} + Y_{CE} = o.$$
$$EA + AC \sin 45^\circ + o = o.$$
$$2{,}000 + AC. \frac{1}{\sqrt{2}} + o = o.$$
$$AC = -2{,}000. \sqrt{2}.$$

Because of the negative result, the Y_{AC} is evidently negative. AC, therefore, acts against the pin; i.e., the member AC is in compression 2,000 $\sqrt{2}$ pounds.

$$\Sigma X = X_{EA} + X_{AC} + X_{CE} = o.$$
$$o - AC \sin 45^\circ + CE = o.$$
$$o - 2{,}000. \sqrt{2}. \frac{1}{\sqrt{2}} + CE = o.$$
$$CE = 2{,}000.$$

The positive result shows that X_{CE} is positive. CE, therefore, acts away from the pin; i.e., the member CE is in tension 2,000 pounds.

Consider the point BFD. The forces acting on the pin are indicated in the Statical Diagram (Fig. 84).

$$\Sigma Y = Y_{BF} + Y_{FD} + Y_{DB} = o.$$
$$BF + o + DB \sin 45^\circ = o.$$
$$2{,}000 + o + DB. \frac{1}{\sqrt{2}} = o.$$
$$DB = -2{,}000. \sqrt{2}.$$

From the negative result, the Y_{DB} is seen to be negative. DB, therefore, acts against the pin; i.e., the member DB is in compression 2,000. $\sqrt{2}$ pounds.

$$\Sigma X = Y_{BF} + Y_{FD} + Y_{DB} = o.$$
$$o + FD + DB \cos 45^\circ = o.$$
$$o + FD + 2{,}000. \sqrt{2}. \frac{1}{\sqrt{2}} = o.$$
$$FD = -2{,}000.$$

The X_{FD} is evidently negative. FD, therefore, acts away from the pin; i.e., the member FD is in tension 2,000 pounds.

Consider the forces acting at the point EFDC. The condition of affairs is represented in the Statical Diagram (Fig. 85).

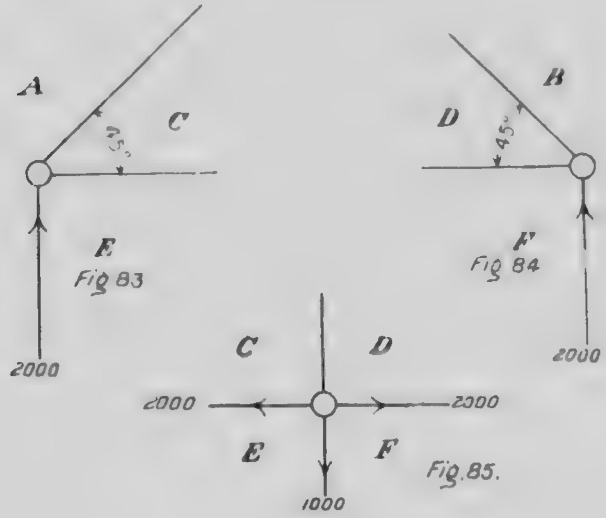


Fig 83

Fig 84

Fig.85.

$$\Sigma Y = Y_{EF} + Y_{FD} + Y_{DC} + Y_{CE} = 0.$$
$$1,000 + 0 + DC + 0 = 0.$$
$$DC = 1,000.$$

From the positive result it is seen that the Y_{DC} is positive. DC, therefore, acts away from the point; i.e., the member DC is in tension 1,000 pounds.

Graphical Solution.

Consider the forces acting at the point EFDC, represented in the Statical Diagram (Fig. 86). Since there are three unknown forces in this case, it is seemingly impossible to construct the Vector Polygon for the set of forces. However, **because FD and CE have the same lines of action, their resultant must act in that line.** Replace FD and CE by their resultant as indicated by R, Statical Diagram (Fig. 87). How this resultant acts is as yet unknown, but, as will be seen, this is of no importance.

Construct the Vector Polygon for the forces shown in the Statical Diagram (Fig. 87). Draw EF (Fig. 88) to represent the known force EF (Fig. 87). Then, from

F (Fig. 88) a line is drawn to represent the known direction of R (Fig. 87). Now, since the set of forces being considered is in equilibrium, we know that from some point in the line drawn to represent the direction of R, another line must be drawn to represent the direction of CD (Fig. 87), and also pass through E

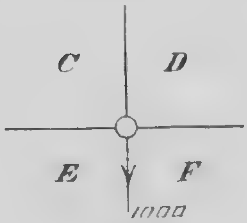


Fig 86

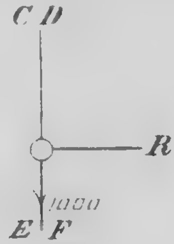


Fig 87

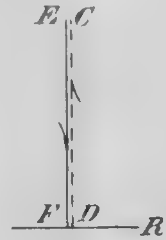


Fig 88.

(Fig. 88), thereby closing the Vector Polygon. It is evident that such a line will coincide with EF (Fig. 88). To avoid confusion, the line representing DC, which would coincide with EF, is shown dotted to one side of EF. The force DC is seen to be an equal and opposite force to EF; i.e., it acts away from the point. The member DC is, therefore, in tension 1,000 pounds.

Consider the forces acting at the point ABDC, Fig. 89 being the Statical Diagram representing the condition of affairs at this point.

AB (Fig. 90) represents the known force AB (Fig. 89), due to the load of 3,000 pounds. Referring to Fig. 89, it is seen that an unknown force intervenes between the force AB and the next known force DC. It will, therefore, be impossible to use Bow's Notation throughout in constructing the Vector Polygon. **Whenever this notation cannot be applied to a line representing any particular force, the line will be shown dotted and the letters designating the force placed beside the line**

From B (Fig. 90) a dotted line is drawn to represent the known force DC. Now, since the forces under consideration are in equilibrium, their Vector Polygon must close, and it must be closed by lines drawn to represent the directions of the unknown forces BD and CA. Therefore, from the termination of the dotted line drawn to represent DC, a dotted line is drawn to represent the direction of the force BD, and from A is drawn another

line to represent the direction of the force CA. These last two lines intersect at C, and it is seen that Bow's Notation applies to the line representing the force CA. This line is, therefore, drawn in full. The Polygon should then read: AB, DC (dotted line), BD (dotted line), and CA.

Having fully determined the unknown forces, it is now possible to go back and construct a Vector Polygon lettered throughout with Bow's Notation.

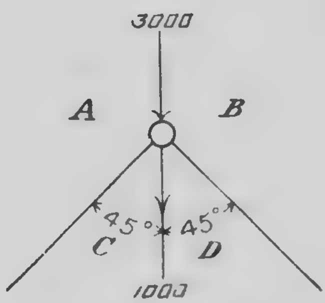


Fig 89.

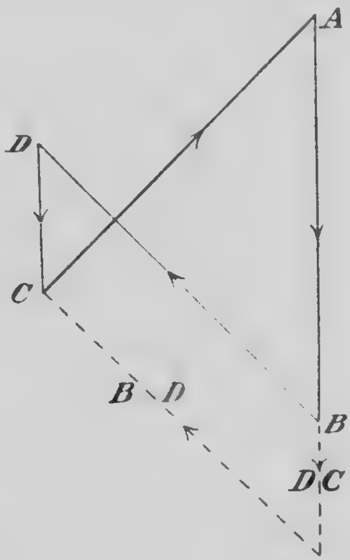


Fig. 90.

From B (Fig. 90) draw the full line BD equal in length and parallel to the dotted line representing the force BD. If D be joined to C, the line DC should then be equal and parallel to the dotted line representing the force DC. This new polygon, which will represent fully the forces acting at the point being considered, reads: AB, BD, DC, and CA.

From either of the last two Vector Polygons it is seen that both the forces BD and CA act against the pin. This places the members BD and CA, respectively, in compression, the magnitudes of the stresses being found from the magnitudes of the lines BD and CA (Fig. 90).

The stresses in the members DF and CE may be found by constructing the Vector Polygons for the points

ACE and BDF, respectively, as shown in Figs. 92 and 94.

Fig. 91 is the Statical Diagram for the forces acting at the point ACE. Constructing the Vector Polygon (Fig. 92) for this set of forces, it is seen that the force CE acts away from the point and the force EA against the point. The member CE is, therefore, in tension, and the abutment reaction EA acts upward as would be expected.

Fig. 94 is the Vector Polygon for the forces acting at the point DBF as represented in the Statical Diagram

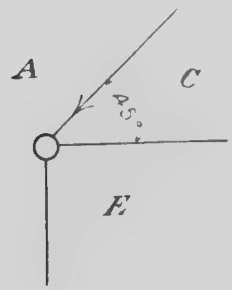


Fig 91

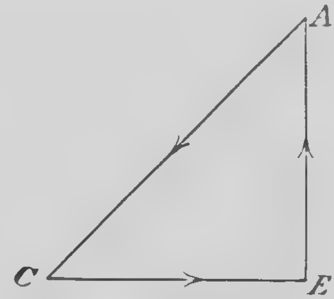


Fig 92

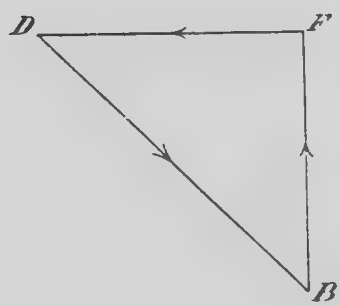


Fig 94

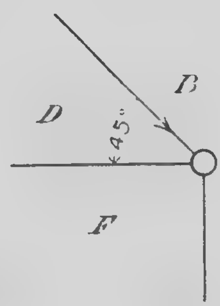


Fig 93

(Fig. 93). The force BF evidently acts against the point and the force FD acts away from the point. The abutment reaction BF, therefore, acts upward, and the member FD is in tension.

Stress Diagram.

The construction of the Stress Diagram for the truss (Fig. 82) is as follows: It will be primarily assumed that the reader has found the stress in the member CD by the method previously indicated. The reason for this assumption is that the construction necessary to find the stress in the member CD will not fit in harmoniously with the subsequent construction of the Stress Diagram.

Considering the point ABDC of the truss (Fig. 82), it is seen that the lines AB, DC (dotted line), BD (dotted line), and CA (Fig. 95) form a Vector Polygon for the forces acting at this point. Reconstructing this polygon by the method outlined in the discussion of the forces indicated in Statical Diagram (Fig. 89), a new Vector

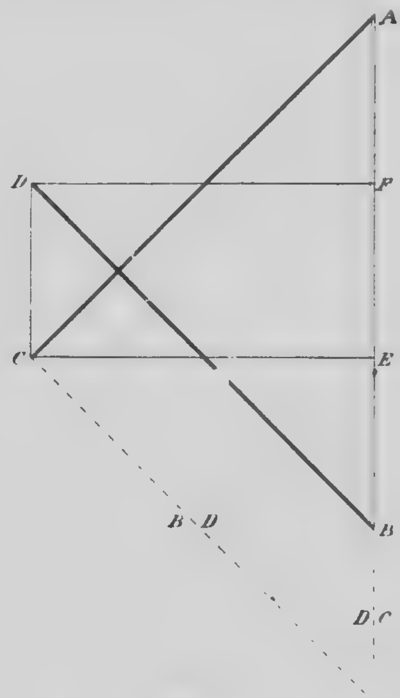


Fig. 95.

Polygon, AB, BD, DC, and CA (Fig. 95), is had, and instead of placing sense marks on the lines composing this last polygon, and any polygons to follow, **a heavy line will be used to indicate forces acting against the point, due to members in compression; a light line indi-**

cating forces acting away from the point, due to tension members; a double line being used to represent force due to loads or abutment reactions, whether they act against or away from the point being considered.

Referring to Statical Diagram (Fig. 91) for the point ACE, it is seen that the force AC exerted by the compression member AC acts against the point, and is necessarily equal but opposite to the force exerted by the same member at the point ABDC. It is evident, then, that the heavy line AC (Fig. 95) may represent the force AC (Fig. 91). If, now, a line be drawn from C to represent the direction of the unknown force CE (Fig. 91), and if a double line be drawn from A to represent the direction of the unknown abutment reaction EA, intersecting the last line at E, the lines AC, CE, and EA will form a Vector Polygon for the forces being considered. It is quite evident that the force EA, being a vertical force, the line EA (Fig. 95) representing it will coincide with the line AB.

Considering the forces acting at the point DBF (Statical Diagram, Fig. 93), it is seen that the heavy line DB (Fig. 95) may represent the force DB (Fig. 93). Drawing from B a double line to represent the direction of the unknown abutment reaction BF (this line will, of course, coincide with BA), and from D a line representing the direction of the unknown force FD, intersecting the double line at F, the Vector Polygon for the forces being considered is completed. This polygon should read DB, BF, and FD. (Compare with Fig. 94)

From a consideration of the point DFEC, it is seen that the lines CD, DF, FE, and EC, already constructed in Fig. 95, form a Vector Polygon for the set of forces acting at this point.

The Queen Post Truss.

The panel widths in the Queen Post Truss (Fig. 97) will be taken as equal. The forces acting on the truss are in equilibrium. Therefore, if in Fig. 97 moments be taken about some point in the line of action of either abutment reaction, and if the equation $\Sigma M = 0$ be applied, the abutment reactions will be found of the magnitudes indicated on the diagram.

Stress Diagram.

Fig. 99 is the Stress Diagram for the truss, Fig. 97. In this diagram, the same system of double, thin, and heavy lines is used as in all the preceding Stress Diagrams drawn.

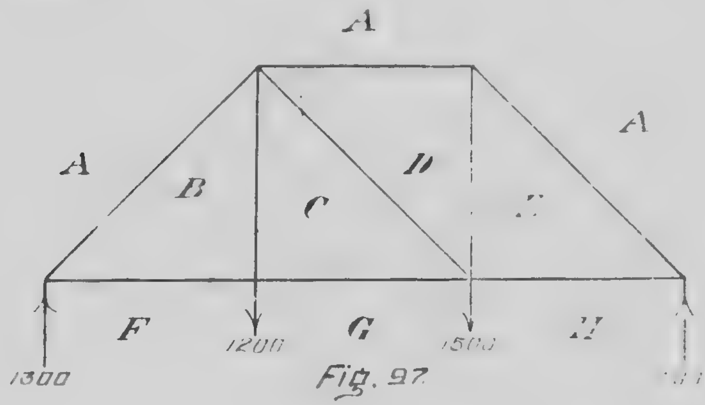


Fig. 97

The reader is advised to construct Fig. 97 on a loose sheet of paper, and, as the stress in each member is found, indicate it on this diagram. This will enable one to see more clearly how the various Statical Diagrams

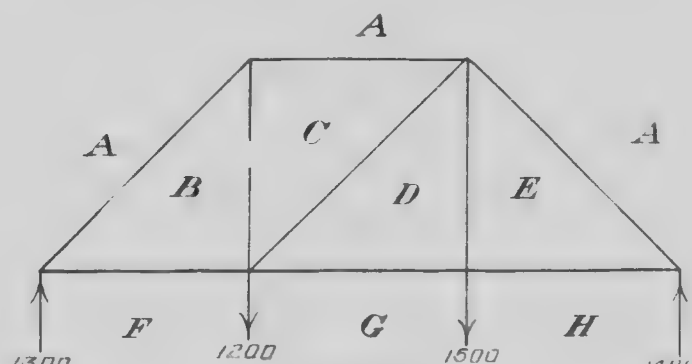


Fig. 98

are arrived at. (It must be kept in mind that the Statical Diagram merely represents the known forces **when the point is first considered.)**

If the following Vector Polygons also be drawn on a separate sheet of paper as they are pointed out in Fig. 99, using the same scale in each case, the construc-

tion of the Stress Diagram will then be more easily followed, for it is generally the presence of lines which have not yet been arrived at that serve to confuse the reader.

Consider the point AFB (Statical Diagram, Fig. 100). AF, FB, BA (Fig. 99) evidently form a Vector Polygon for the forces acting at this point. From this polygon, the force FB is seen to act away from the point

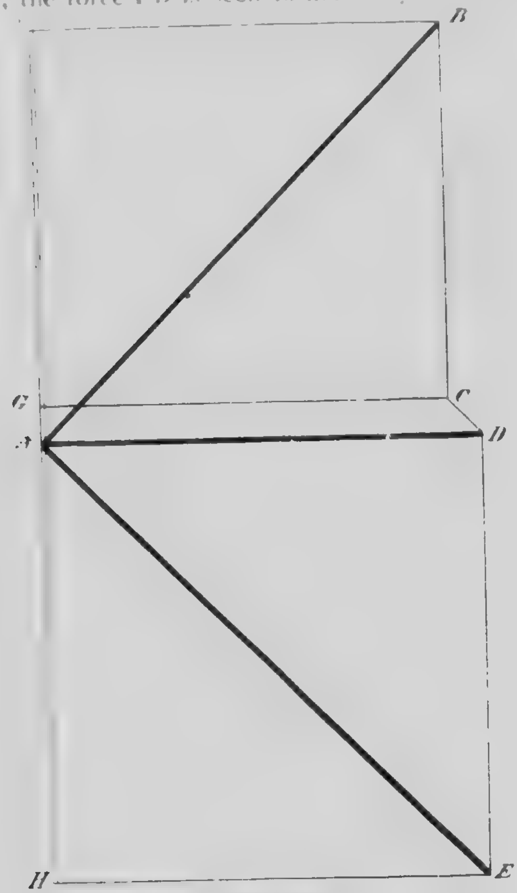


Fig. 99.

and BA against the point. The member FB is, therefore, in tension and the member BA in compression. (The magnitudes of these stresses will, of course, be given by scaling the lengths of the lines FB and BA.)

Consider the point BFGC (Statical Diagram, Fig. 101).

The known force BF (Fig. 101) is exerted by the tension member BF (or FB). This force, is, therefore, equal and opposite to the force exerted by the same member at the point AFB. But FB (Fig. 99) represents the force exerted at the point AFB; therefore, the same line read BF will represent the force BF exerted at the point BFGC. The line FG (Fig. 99) represents the load FG. (The load FG is less than the reaction AF.) The lines GC and CB complete the polygon, which should then read: BF, FG, GC, and CB. The members GC and CB are evidently both in tension.

Consider the point ABCD (Statical Diagram, Fig. 102).

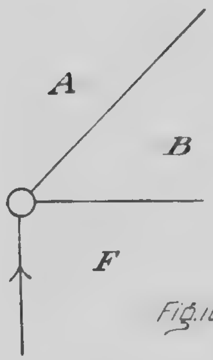


Fig. 100

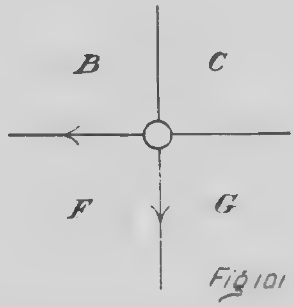


Fig. 101

The Vector Polygon for this point reads on Fig. 99: AB, BC, CD, and DA, from which it is seen that the force CD acts away from the point and the force DA against the point. The member CD is, therefore, in tension and the member DA in compression.

Consider the forces acting at the point ADE (Statical Diagram, Fig. 103). Referring to Fig. 99, the Vector Polygon for these forces reads: AD, DE, and EA, from which it is seen that the member DE is in tension and the member EA in compression.

Finally, **from a consideration of the point EDCGH** (Statical Diagram, Fig. 104), the member HE is found to be in tension. The Vector Polygon for this point reads on Fig. 99: ED, DC, CG, GH (constructed to represent the load of 1,500 pounds), and HE. (The same result would have been arrived at had the point AEH been considered.)

Fig. 98 represents a Truss, on which the loading is identical to that on the truss, Fig. 97. The Stress Diagram for this last truss is shown at Fig. 105, the construction of which may be seen by considering the various points of the truss in the following order: AFB, ABC, CBFGD, ACDE, EDGH.

It is important to notice that the stress in the member CD (Fig. 97) is tension, whereas the member CD (Fig. 98) is in compression, but the magnitude of the stress is, in both cases, the same. That is, **provided the load be kept constant in magnitude and position,** the change in inclination of the member CD merely changes the nature of the stress, but not its magnitude.

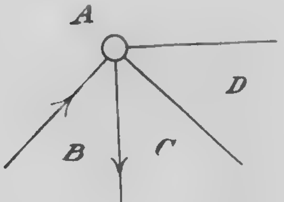


Fig 102

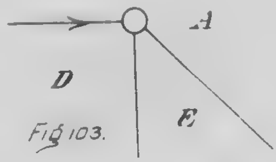


Fig 103.

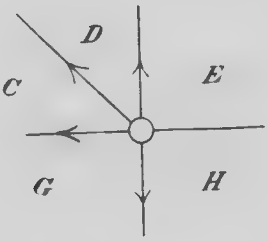


Fig 104.

It may be pointed out here that on account of the construction of the joints, the diagonals in the old Queen Post Trusses were merely capable of taking up compression, yet we have just seen that if the truss were built as shown in Fig. 97, the member CD would be in tension for the given load. This difficulty is overcome by placing a second diagonal between the remaining corners of the panel. Such a diagonal is known as a counter brace, the theory of which will be taken up under the discussion of the Pratt Truss.

The French Roof Truss.

Fig. 106 represents a form of roof truss known as the French Truss. Such a structure is generally employed where the roof pitch is steep and where overhead space is desired; for example, in a church. Fig. 107 shows how the same idea may be further carried out for a larger truss.

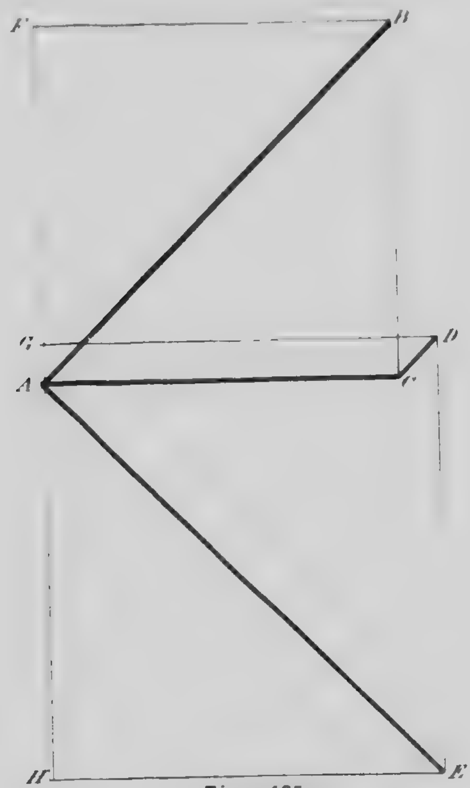


Fig. 105.

The Fink Roof Truss.

The Fink Truss (Fig. 108) is much more commonly used than the French truss, of which it is an evolution. Wherever the roof pitch is low, as in factories, machine shops, etc., it is readily seen that this truss is much more adaptable than one of the form shown in Figs. 106 and 107.

The method of determining the stresses in the members of either the French or Fink truss is exactly

the same, so that it will only be necessary to indicate the procedure in one case.

In order to simplify matters as much as possible, the inclination of the roof to the horizontal will be taken as 30° in the truss (Fig. 108), which is rather a steeper pitch than is ordinarily used.

Analytical Solution.

Considering the truss as a body acted upon by a set of exterior forces which are in equilibrium, and applying the equation $\Sigma M = 0$ to these forces, it is found by taking

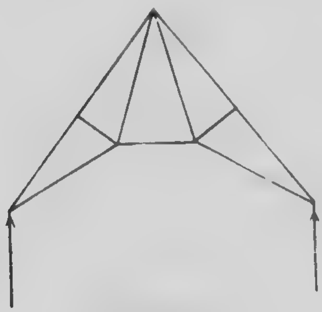


Fig. 106.

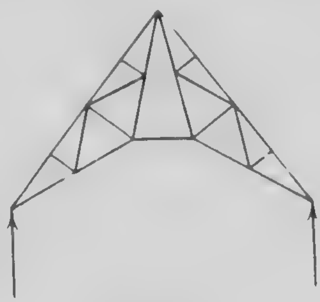


Fig. 107.

moments about a point in either abutment reaction, that the abutment reactions are both equal to half the total load.

Effective Loads and Abutment Reactions.

It will be noticed that no load is indicated at the joint over each abutment. In reality, there is a load transferred to the truss at each of these positions due to the fact that a purlin would be placed over these joints (see Fig. 82 A). However, a load at either of these joints causes no stress in the members of the truss, but is, as it were, transferred directly to the abutment, thereby only causing a reaction equal and opposite to itself. These loads may, therefore, be left out of consideration. The abutment reactions found by considering the remaining loads **(the effective loads)** may in consequence be called **the effective abutment reactions**, since they are not the total reactions. If this is not quite clear to the reader, it would be advisable for him to place in loads over the joints referred to, and then after determining the **total abutment reactions**, proceed to find the stresses in the

various members of the truss. These stresses should be exactly the same as the stresses found by considering merely the effective abutment reactions and loads.

The truss being symmetrical in construction, **it will only be necessary to determine the stresses in the left-**

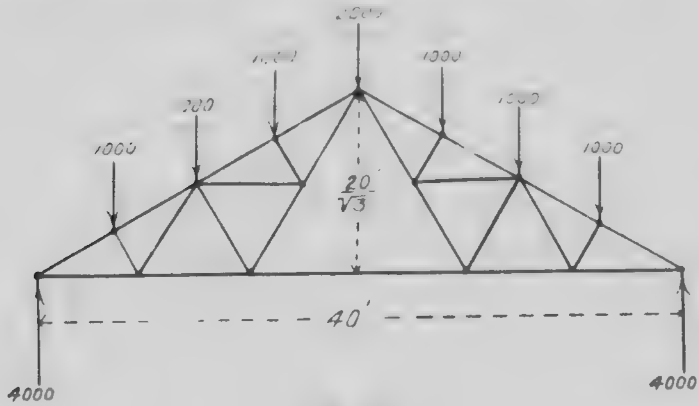


Fig. 108.

hand half of the truss, for, since the loading is also symmetrical, the stresses in corresponding members of both halves of the truss will be the same.

If the algebraic solution be carried on for the truss in its ordinary position, it will be found that the work is

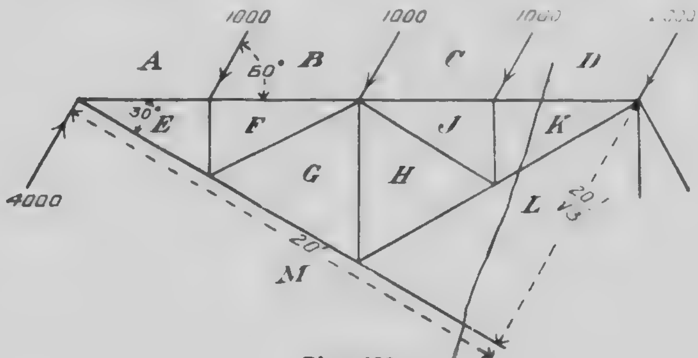


Fig. 109.

extremely tedious on account of the various inclinations of the members. This difficulty may be gotten over, to a certain extent, by considering the truss as turned over till the left-hand upper chord members lie in the horizontal as indicated in Fig. 109. (Merely the left-hand

half of the truss is shown. This artifice cannot in any way affect the stresses in the members, since the relative lines of action of the forces acting on the truss (loads, etc.), remain unchanged.

Consider the Point AME (Statical Diagram, Fig. 110)

$$\Sigma Y = Y_{AM} + Y_{ME} + Y_{EA} = 0.$$

$$4,000 \sin 60^\circ + ME. \sin 30^\circ + 0 = 0.$$

$$4,000.\frac{\sqrt{3}}{2} + ME.\frac{1}{2} = 0.$$

$$ME = -4,000\sqrt{3}.$$

From the negative result, the assumption as to Y_{ME} is seen to be wrong. Y_{ME} is, therefore, negative; i.e., ME acts away from the point. The member ME is in tension $4,000\sqrt{3}$ pounds.

$$\Sigma X = X_{AM} + X_{ME} + X_{EA} = 0.$$

$$4,000 \cos 60^\circ + 4,000\sqrt{3}. \cos 30^\circ + EA = 0.$$

$$4,000.\frac{1}{2} + 4,000\sqrt{3}.\frac{\sqrt{3}}{2} + EA = 0.$$

$$EA = -8,000.$$

The assumption as to X_{EA} is evidently wrong (negative result). X_{EA} is negative; i.e., EA acts to the left against the point. The member EA is, therefore, in compression 8,000 pounds.

Consider the Point BAEF (Statical Diagram, Fig. 111)

$$\Sigma Y = Y_{BA} + Y_{AE} + Y_{EF} + Y_{FB} = 0.$$

$$-1,000 \sin 60^\circ + 0 + EF + 0 = 0.$$

$$EF = 500\sqrt{3}.$$

Y_{EF} is evidently positive. EF, therefore, acts upward against the point; i.e., the member EF is in compression $500\sqrt{3}$ pounds.

$$\Sigma X = X_{BA} + X_{AE} + X_{EF} + X_{FB} = 0.$$

$$1,000 \cos 60^\circ + 8,000 + 0 + FB = 0.$$

$$1,000.\frac{1}{2} + 8,000 + FB = 0.$$

$$FB = -7,500.$$

X_{FB} being found negative, FB must act to the left against the point; i.e., the member FB is in compression 7,500 pounds.

Consider the Point FEMG (Statical Diagram, Fig. 11-).

The equations $\Sigma X = 0$ and $\Sigma Y = 0$ will have to be used simultaneously at this point

$$\Sigma X = X_{FE} - X_{EM} + X_{MG} + X_{GF} = 0$$

$$- 4,000 \sqrt{3}. \cos 30^\circ + MG \cos 30^\circ + GF \cos 30^\circ = 0$$

Cancelling out cos 30°

$$MG = 4,000 \sqrt{3} - GF \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

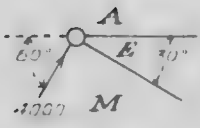


Fig 110

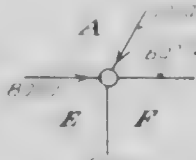


Fig 111

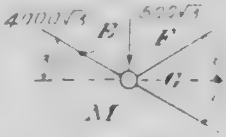


Fig 112

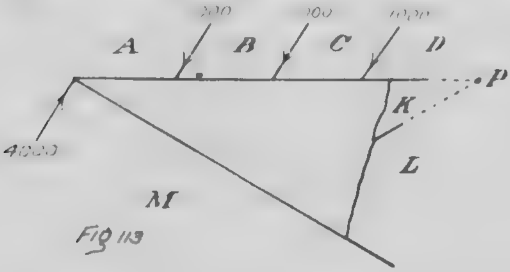


Fig 113

In accordance with the X_{MG} having been assumed positive, the Y_{MG} must be assumed negative if the equations are to be used simultaneously. In the same way, from a consideration of the line of action of GF, the Y_{GF} must be assumed positive if the X_{GF} be assumed positive.

$$\Sigma Y = Y_{FE} + Y_{EM} + Y_{MG} + Y_{GF} = 0.$$

$$- 500 \sqrt{3} + 4,000 \sqrt{3}. \sin 30^\circ - MG. \sin 30^\circ + GF \sin 30^\circ = 0.$$

$$- 500 \sqrt{3} + 4,000 \sqrt{3}. \frac{1}{2} - MG. \frac{1}{2} + GF. \frac{1}{2} = 0 \ldots\ldots (2.)$$

Substitute value of MG from (1.) into (2.)

$$- 500 \sqrt{3} + 2,000 \sqrt{3} - (4,000 \sqrt{3} - GF). \tfrac{1}{2} + GF. \tfrac{1}{2} = 0.$$

$$- 500 \sqrt{3} + 2,000 \sqrt{3} - 2,000 \sqrt{3} + GF. \tfrac{1}{2} + GF \tfrac{1}{2} = 0.$$

$$GF = 500 \sqrt{3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

The positive value for GF shows that the assumptions as to the X and Y of GF were correct. Since X and

Y of GF were both assumed positive, GF must act away from the point. The member GF is, therefore, in tension $500\sqrt{3}$ pounds.

Substituting the value of GF from (3.) into (1.)

$$MG = 4{,}000\sqrt{3} - 500\sqrt{3}.$$
$$= 3{,}500\sqrt{3}.$$

The assumptions as to X_{MG} and Y_{MG} are evidently correct (positive result). MG, therefore, acts away from the point (X assumed positive and Y negative). The member MG is in tension $3{,}500\sqrt{3}$ pounds.

Considering any of the remaining points, as the problem now stands, it is seen that it is impossible to determine the stress in all of the remaining members by the equations $\Sigma X = 0$ and $\Sigma Y = 0$, since there are found to be three unknown or more at each point.

It is possible, however, to find the stress in the member ML by the Method of Sections.

Take a section through the members ML, LK, and KD.

Consider the forces acting on the portion of the truss to the left of the section. In order to eliminate the lines representing the various members of the truss, which sometimes tend to confuse one, the body to the left of the section may be considered as solid, since this will in no way alter the relative lines of action and magnitudes of the forces acting on the body (see Fig. 113).

The forces acting on this body are in equilibrium. Therefore, $\Sigma M = 0$.

Take moments about the point P (the intersection of LK and KD).

$$\Sigma M = M_{AM} + M_{ML} + M_{LK} + M_{KD} + M_{DC} + M_{CB} + M_{BA} = 0.$$

$$4{,}000.20 + ML.\frac{20}{\sqrt{3}} + 0 + 0 - 1{,}000.5 - 1{,}000.10 - 1{,}000.15 = 0.$$

$$ML = -50{,}000\cdot\frac{\sqrt{3}}{20}.$$
$$= -2{,}500\sqrt{3}.$$

From the negative result we see that the M_{ML} about P is negative. ML must, therefore, act away from the

section; i.e., the member ML is in tension $2,500\sqrt{3}$ pounds.

Consider the Point GMLH.

$$\Sigma X = X_{GM} + X_{ML} + X_{LH} + X_{HG} = 0.$$

$$-3,500\sqrt{3}.\cos 30^\circ + 2,500\sqrt{3}.\cos 30^\circ + LH\cos 30^\circ + 0 = 0.$$

$$LH = 1,000\sqrt{3}.$$

The member LH is evidently in tension $1,000\sqrt{3}$ pounds.

$$\Sigma Y = Y_{GM} + Y_{ML} + Y_{LH} + Y_{HG} = 0.$$

$$3,500\sqrt{3}.\sin 30^\circ - 2,500\sqrt{3}.\sin 30^\circ + 1,000\sqrt{3}.\sin 30^\circ + HG = 0.$$

$$\tfrac{1}{2}(3,500\sqrt{3} - 2,500\sqrt{3} + 1,000\sqrt{3}) + HG = 0.$$

$$HG = -1,000\sqrt{3}.$$

The member HG is in compression $1,000\sqrt{3}$ pounds.

By considering the points CBFGHJ, DCJK, and KJHL, the stresses in the remaining members may be found.

Graphical Solution.

It is advisable in this problem to first construct a separate Vector Polygon for each set of forces considered, and then draw the Stress Diagram rather than try to follow the Vector Polygons on a Stress Diagram, as was done in the case of the Queen Post Truss.

Assuming that the effective Abutment Reactions have been found, Fig. 114 is the Statical Diagram for **the point MAE,** Fig. 115 being the corresponding Vector Polygon from which it is seen that the force AE acts

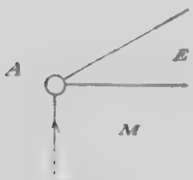
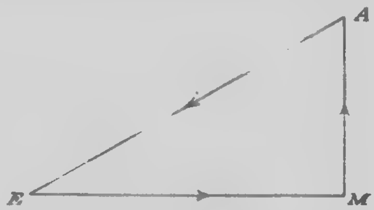


Fig 114



Fig

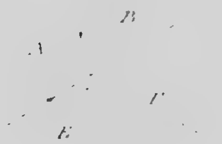
against the point and the force EM away from the point. The magnitudes of the forces AE and EM will be given by scaling AE and EM (Fig. 115). The member AE

is, therefore, in Compression and the member EM in Tension.

In the following discussion it will be merely necessary to show whether the various members are in tension or compression, since finding the magnitudes of these stresses from the figures can only be done by either scaling the lengths of the lines; which is impracticable, for the printed polygons are not of a size suitable to any particular scale; or, by applying geometrical reasoning to the figures, which may easily be done by the reader.

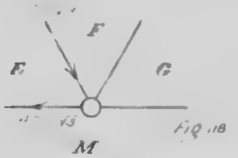
It is, of course, always best for the reader to construct the polygons or figures for himself, using some convenient scale.

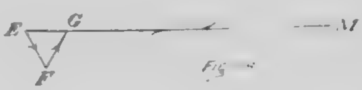
Consider the point EABF. Fig. 116 is the Statical Diagram for the forces acting at this point. From the Vector Polygon (Fig. 117) it is seen that the forces BF



and FE both act against the point. The members BF and FE are, therefore, in Compression.

Considering the forces acting at the point MEFG (Statical Diagrams, Fig. 118). From the Vector Polygon (Fig. 119), it is seen that both the forces FG and GM act away from the point, the members FG and GM being, in consequence, in Tension. (The reader will





notice that the Vector Polygon reads ME, EF, FG, and GM. This last line GM coincides with the first line ME, the sense mark of ME having been placed above and that of GM below.)

Consider, next, the point CDKJ (Statical Diagram, Fig. 120). The forces acting at this point are CD

(known), and DK, KJ, and JC, all unknown. Since there are three unknowns at this point, it is impossible to construct a Vector Polygon as the problem now stands. JC and DK, however, have the same lines of action. Their resultant will, therefore, act in this same line.

Replace JC and DK by their resultant R_1 as indicated in Statical Diagram, Fig. 121. Constructing the Vector Polygon for this new set of forces, which only contains two unknown, it is seen from Fig. 122 that the force KJ acts against the point. The member KJ

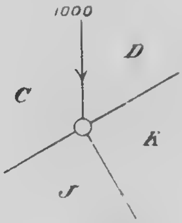


Fig 120.

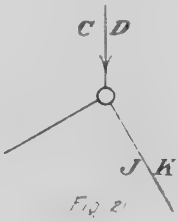


Fig 121

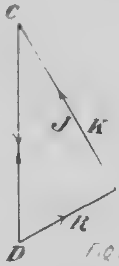


Fig 122

is, therefore, in Compression. (The magnitude of the resultant R_1 gives no information of value.)

Consider the point JKLH (Statical Diagram, Fig. 123). Since KL and LH also have the same line of action, their resultant will act in that line. Replacing KL and LH by their resultant R_2, a condition such as shown in Statical Diagram, Fig. 124, is arrived at. Con-

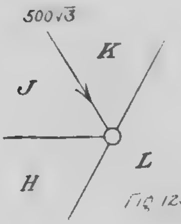


Fig 123

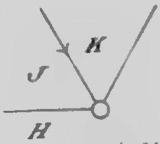
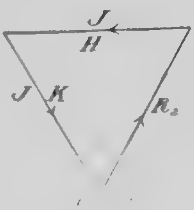


Fig 124



structing the Vector Polygon for this last set of forces, it is seen from Fig. 125 that HJ acts away from the point, thereby showing that the member HJ is in Tension.

Coming back to the point GFBCJH (Statical Diagram, Fig. 126), it is seen that there are now merely

two unknowns, so that the Vector Polygon for the forces acting at this point may be constructed.

The Vector Polygon (Fig. 127) may be begun by representing the forces GF, FB, and BC, but it is then seen that the unknown force CJ intervenes between BC and the next known force JH; so that, for the time being, Bow's Notation will have to be dropped.

From C (Fig. 127) draw a dotted line to represent the known force JH, placing the letters J and H beside

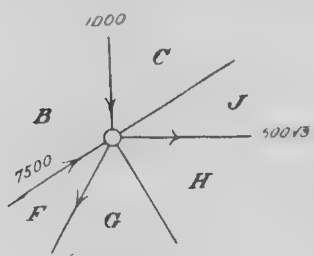


Fig 125

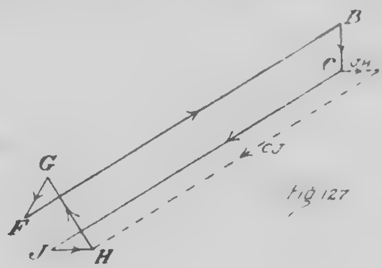


Fig 127

the line to indicate which force it represents. From the termination of this last dotted line is drawn another dotted line to represent the known line of action of the force CJ. Also, since the Vector Polygon must close, a line is drawn from the initial point G to represent the known line of action of the force HG, this last line intersecting the second dotted line CJ at H. The Vector Polygon should then read: GF, FB, BC, JH (dotted line), CJ (dotted line), and HG. It is evident that Bow's Notation will apply to the line representing the force HG, therefore, the line is drawn in full.

Now that all the forces are known at the point being considered, it is possible to go back and reconstruct the Vector Polygon, using Bow's Notation throughout.

From C (Fig. 127) draw CJ equal and parallel to the dotted line CJ. The line joining J to H is then equal and parallel to the dotted line JH. It is evident that these new lines, CJ and JH, represent fully the forces CJ and JH.

The new Vector Polygon reads: GF, FB, BC, CJ, JH, and HG, from which it is seen that the forces CJ and HG both act against the point. The members CJ and HG are evidently both in compression. (This same conclusion could have been arrived at from the first Vector Polygon, but, as will be seen later, it is necessary

to reconstruct the polygon, using Bow's Notation if the Stress Diagram is to be drawn.)

Stress Diagram.

Fig. 128 is the complete Stress Diagram for the left-hand half of the Fink Truss as shown in Fig. 128A. It will merely be necessary to point out the various Vector Polygons in this diagram since the construction of these polygons has been already gone through.

MA, AE, and EM (Fig. 128) evidently form a Vector Polygon for the forces acting at **the point MAE.** (See Figs. 114 and 115.)

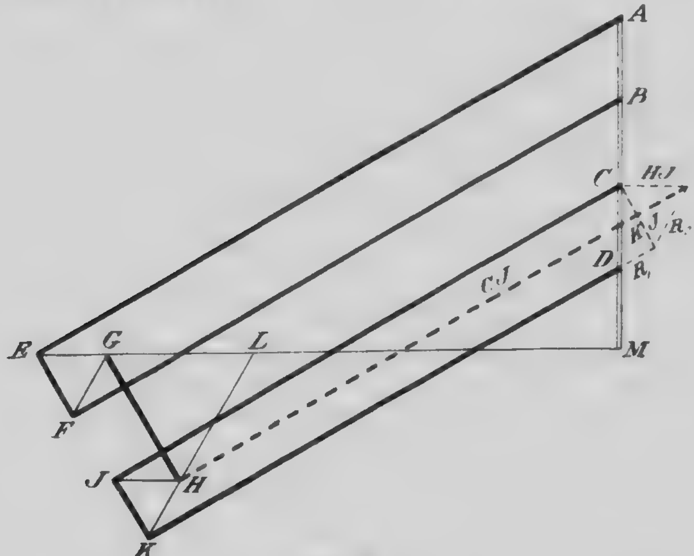


Fig. 128.

Considering **the point EABF,** the Vector Polygon reads on Fig. 128: EA, AB, BF, and FE. (Compare with Fig. 117.)

Coming next to **the point MEFG** (see Figs. 118 and 119), ME, EF, FG, and GM (Fig. 128) form a Vector Polygon for the forces acting at this point.

From B, on the line MA, lay off BC to represent the load BC. Then from C, on the same line, lay off CD to represent the load CD.

Consider **the point KJCD.** Replacing DK and JC by their resultant R_1, it is seen that CD, R_1 (dotted), and KJ (dotted) form the Vector Polgon for the forces indicated in Statical Diagram, Fig. 121.

Following out the same method for **the point JKLH**, replacing KL and LH by their resultant R_2, which gives a condition indicated in Statical Diagram (Fig. 124), it is evident that JK (dotted), R_2 (dotted), and HJ (dotted) (Fig. 128) form the Vector Polygon for the forces being discussed. (Compare with Fig. 125.)

From a consideration of the forces acting at **the point GFBCJH**, it is seen that GF, FB, BC, JH (dotted line), CJ (dotted line), and HG (Fig. 128) form a Vector Polygon for the forces being discussed. Bow's Notation, however, could not be fully carried out for reasons already given.

Reconstructing the Vector Polygon for this point so that Bow's Notation may be applied throughout, a polygon which reads GF, FB, BC, CJ (heavy full line), JH (thin full line), and HG (in Fig. 128) is arrived at. (See discussion of Figs. 126 and 127.)

In the discussion of the Graphical Solution previous to this section on the Stress Diagram, the determination of the stress in the truss members was not carried beyond

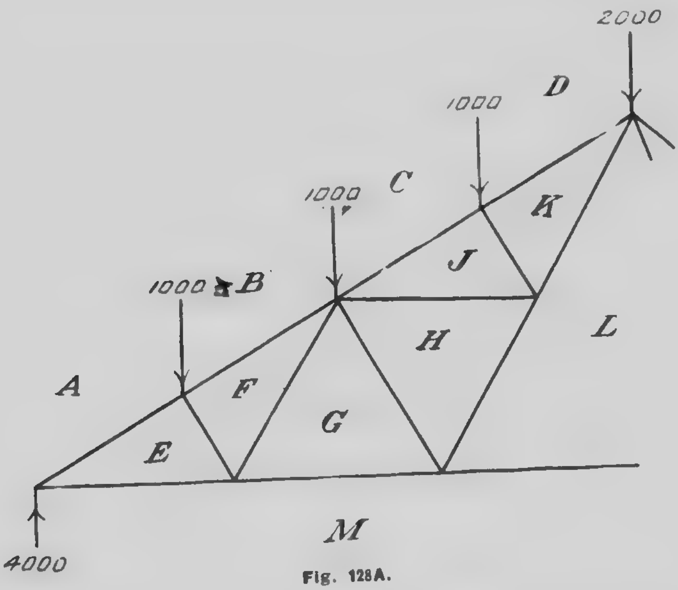


Fig. 128A.

the point GFBCJH, there being nothing difficult in the constructing of the Vector Polygons for the forces acting at the remaining points. These polygons may easily be followed out on the Stress Diagram as follows:—

Referring to Statical Diagram (Fig. 129) for **the point MGHL**, it is seen that MG, GH, HL, and LM (Fig. 128) constitute a Vector Polygon for the forces acting at this point. It is evident from this polygon that both the forces HL and LM act away from the point. The members HL and LM are, therefore, both in Tension.

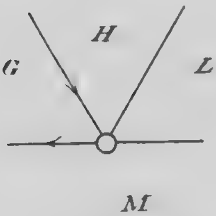


Fig 129

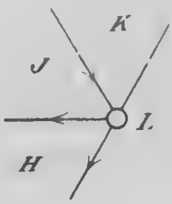


Fig 130

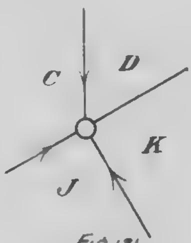


Fig 131

Consider **the point LHJK** (Statical Diagram, Fig. 130). LH, HJ, JK, and KL (Fig. 128) is the Vector Polygon for the forces acting at this point, from which it is seen that the member KL is in Tension.

Coming back, finally, to **the point KJCD** (Statical Diagram, Fig. 131), evidently KJ, JC, CD, and DK (Fig. 128) form a Vector Polygon for the forces being considered. The force DK is seen to act against the point. The member DK must, in consequence, be in Compression.

The Pratt Truss.

Fig. 133 represents a form of bridge truss known as a Pratt Truss. The Effective Dead Load shown in the diagram has been chosen purely arbitrarily without regard to the practical rules which govern the first assumptions as to dead loads on bridges—convenient quantities having been chosen which will serve to illustrate the method of procedure.

Since the truss (Fig. 133) is symmetrically constructed and loaded, it will merely be necessary to consider one-half of the structure, for corresponding members in both halves will have the same stresses.

The required stresses may be determined by either considering the forces acting on the pin at each joint of the truss, or the Method of Sections may be applied by taking sections through various members in such a

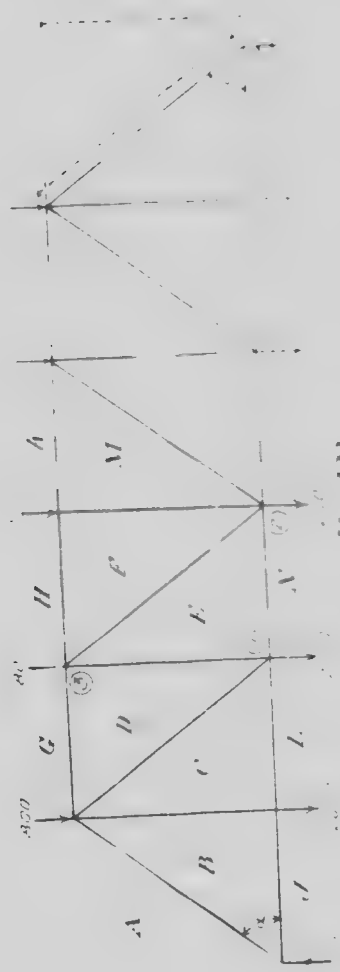


Fig. 133

Consider the forces acting on the pin at the joint **BAJ** (Statical Diagram, Fig. 134).

$$\Sigma Y = Y_{AJ} + Y_{JB} + Y_{BA} = 0.$$

$$7{,}000 + 0 + BA.\ 4/5 = 0.$$

$$BA = -5/4 \cdot 7{,}000 = -8{,}750.$$

The Y of BA is evidently negative. The force BA, therefore, acts against the point: i.e., the member BA is in Compression 8,750 pounds.

$$\Sigma X = X_{AJ} + X_{JB} + X_{BA} = 0.$$

$$O + JB - 8{,}750 \cdot 3/5 = 0.$$

$$JB = 5{,}250.$$

The positive result shows that X_{JB} is positive; i.e., JB acts to the right away from the point. The member JB is, therefore, in Tension 5,250 pounds.

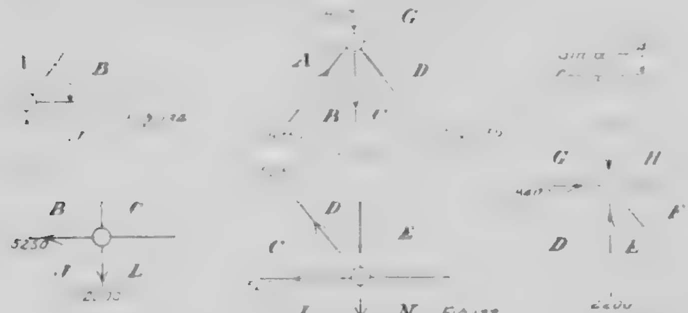


Fig 135 Fig 137 Fig 138

Fig. 135 is the Statical Diagram for the forces acting at the point LJBC.

$$\Sigma X = X_{LJ} + X_{JB} + X_{BC} + X_{CL} = 0.$$

$$O - 5{,}250 + O + CL = 0.$$

$$CL = 5{,}250.$$

The member CL is evidently in Tension 5,250 pounds.

$$\Sigma Y = Y_{LJ} + Y_{JB} + Y_{BC} + Y_{CL} = 0.$$

$$-2{,}000 + O + BC + O = 0.$$

$$BC = 2{,}000.$$

The member BC is in Tension 2,000 pounds.

For the point CBAGD (Statical Diagram, Fig. 136).

$$\Sigma Y = Y_{CB} + Y_{BA} + Y_{AG} + Y_{GD} + Y_{DC} = 0.$$

$$-2{,}000 + 8{,}750 \cdot 4/5 - 800 + O + DC \cdot 4/5 = 0.$$

$$DC = -5{,}250.$$

The member DC is in Tension 5,250 pounds.

$$\Sigma X = X_{CB} + X_{BA} + X_{AG} + X_{GD} + X_{DC} = 0.$$

$$O + 8{,}750 \cdot 3/5 + O + GD + 5{,}250 \cdot 3/5 = 0.$$

$$GD = -8{,}400.$$

The member GD is in Compression 8,400 pounds.

Considering the forces acting at the point NLCDE (Statical Diagram, Fig. 137):—

$$\Sigma X = X_{NL} + X_{LC} + X_{CD} + X_{DE} + X_{EN} = 0.$$
$$O - 5{,}250 - 5{,}250 \cdot 3/5 + O + EN = 0.$$
$$EN = 8{,}400.$$

The member EN is in Tension 8,400 pounds.

$$\Sigma Y = Y_{NL} + Y_{LC} + Y_{CD} + Y_{DE} + Y_{EN} = 0.$$
$$-2{,}000 + O + 5{,}250 \cdot 4/5 + DE + O = 0.$$
$$DE = -2{,}200.$$

The member DE is in Compression 2,200 pounds.

Statical Diagram (Fig. 138) represents the condition at the point EDGHF.

$$\Sigma Y = Y_{ED} + Y_{DG} + Y_{GH} + Y_{HF} + Y_{FE} = 0.$$
$$2{,}200 + O - 800 + O + FE \cdot 4/5 = 0.$$
$$FE = -1{,}750.$$

The member FE is in Tension 1,750 pounds.

$$\Sigma X = X_{ED} + X_{DG} + X_{GH} + X_{HF} + X_{FE} = 0.$$
$$O + 8{,}400 + O + HF + 1{,}750 \cdot 3/5 = 0.$$
$$HF = -9{,}450.$$

The member HF is in Compression 9,450 pounds.

From inspection of the point FHKM, it is easily seen that the member FM must be in Compression 800 pounds.

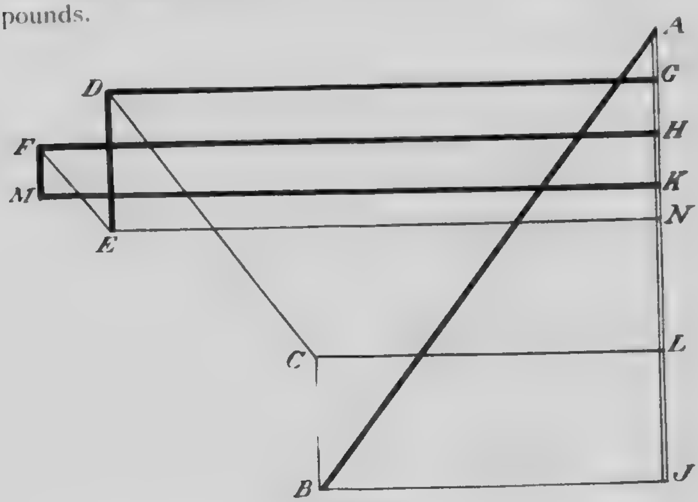


Fig. 139

Stress Diagram.

The Stress Diagram for the Pratt Truss, loaded as indicated in Fig. 133, is shown at Fig. 139. If the sets

of forces acting at the various joints of the truss be considered in the same order as taken up in the preceding analytical solution (see Statical Diagrams, Figs 134 . . . 138), the Vector Polygons for the respective cases may easily be followed out on the diagram.

Live Load Stresses.

A few structural details will have to be pointed out in order that the reader may clearly understand the following discussion of Live Load stresses:--

Fig. 133 represents the truss on one side of a bridge. On the other side is a truss of the same construction. Connecting these trusses at the top is a system of bracing, and at the lower joints provision is made to

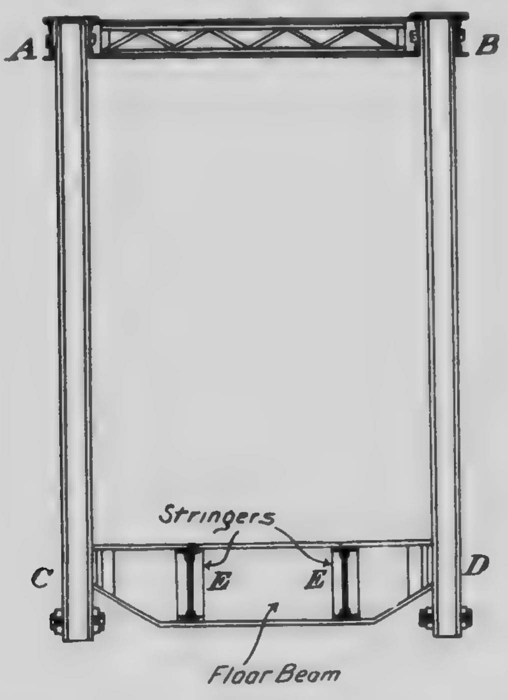


Fig 140

support the floor system. Fig. 140 represents diagrammatically by an end elevation, the method of supporting the flooring, etc. This figure represents an end elevation of the joints (1.) and (3.) (Fig. 133) and the corresponding joints on the opposite truss of the bridge.

AC and BD represent the vertical [illegible] between A and B. Riveted to the lower [illegible] of these vertical members is a beam, fastened [illegible] are as many other longitudinal stringers (EE) [illegible] be thought desirable to support the floor upon which the Live Load will pass. In this manner, the Live Load is transferred by the stringers EE to the floor beam placed between verticals at C and D, and from thence to the joints of the truss.

If now, a Live Load be in some position between the lower joints (1.) and (2.) (Fig. 133), it is seen from the preceding discussion that part of the load will be transferred to the joint (1.) and part to the joint (2.), the portion of the load transferred to either joint depending upon the distance of the load from (1.) or (2.)

It is seen from the preceding discussion on the construction of the floor beams and stringers that the live load is transferred by these beams, etc., to the joints of the truss. If a Live Load occupies a position between two joints, part of the load goes to one joint and part to the other joint, so that if the stress in any particular member is desired, **the truss must be considered as acted upon by the portions of the load transferred to the respective joints between which the load lies.** For instance, if the load were to lie half way between the joints (2) and (3) (Fig. 141), half of the load goes to (2) and half to (3). The truss should then be considered as acted upon by two forces, one at (2) and the other at (3), and, in this case, both equal to half the load.

It is merely necessary, for the time being, to consider the Live Load as it comes directly over each joint, in which case the total load is taken as acting on the particular joint at which it lies.

Fig. 141.

In Fig. 141 the dotted line represents the level along which the Live Load of 6,000 pounds passes.

Consider the load at joint (2). T[illegible]

[illegible]

[illegible] the load is [illegible]

[illegible]

Take a section through the members AB and BJ. Consider the forces acting on the portion of the truss to the left of this section. (This portion to the left may be considered as a solid body.) The forces acting on this body are in equilibrium, and act as represented in Fig. 142.

$$\Sigma Y = Y_{JA} + Y_{AB} + Y_{BJ} = 0.$$
$$5{,}000 + 4/5\ AB + 0 = 0.$$
$$AB = -6{,}250.$$

AB is evidently in Compression 6,250 pounds.

Consider the forces acting on the portion of the truss to the left of a section through the members AD, DC, and CK. In this case, and all of the following cases, the area underneath the lower chords to the left of the load will be lettered J, and the area to the right of the load lettered K. It is seen that these areas will change whenever the load takes up a new position. (See Fig. 143.)

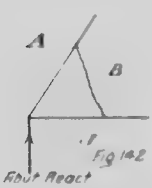


Fig 142

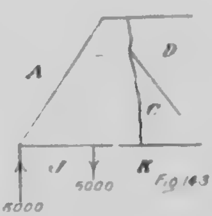


Fig 143

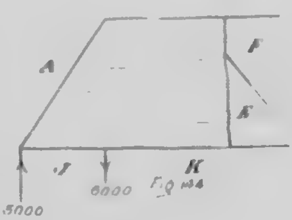


Fig 144

$$\Sigma Y = Y_{JA} + Y_{AD} + Y_{DC} + Y_{CK} + Y_{KJ} = 0.$$
$$5{,}000 - 0 + DC . 4/5 + 0 - 6{,}000 = 0.$$
$$DC = 5/4 . 1{,}000.$$
$$= 1{,}250.$$

The member DC is in Compression 1,250 pounds.

By taking a section through the members AF, FE, and EK and considering the forces acting on the truss to the left of this section (see Fig. 144), the member FE may also be shown to be in Compression 1,250 pounds.

Consider the load at joint (3). The left-hand abutment reaction in this case is $\frac{2}{3}$. 6,000 = 4,000 pounds.

Considering the forces acting on the truss to the left of a section through AB and BJ, it is seen that Fig. 142 will again represent the condition of affairs, provided 4,000 be substituted as the value of the abutment reaction.

$$\Sigma Y = Y_{JA} + Y_{AB} + Y_{BJ} = 0.$$
$$4{,}000 + AB\ \tfrac{4}{5} + 0 = 0.$$
$$AB = -\tfrac{5}{4} \cdot 4{,}000.$$
$$= -5{,}000.$$

AB in this case is in Compression 5,000 pounds.

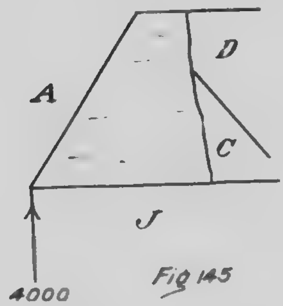


Fig 145

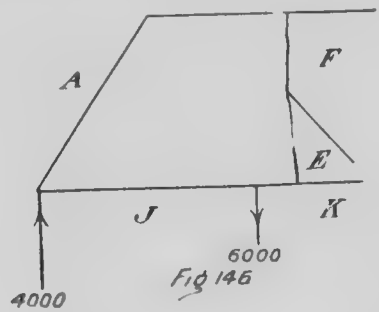


Fig 146

Consider the forces acting on the portion of the truss to the left of a section through AD, DC, and CJ. (See Fig. 145.)

$$\Sigma Y = Y_{JA} + Y_{AD} + Y_{DC} + Y_{CJ} = 0.$$
$$4{,}000 + 0 + DC.\ \tfrac{4}{5} + 0 = 0.$$
$$DC = -\frac{5}{4}.\ 4{,}000.$$
$$= -5{,}000.$$

The member DC is in Tension 5,000 pounds.

Fig. 146 represents the forces acting on the portion of the truss to the left of a section through AF, FE,

$$\Sigma Y = Y_{JA} + Y_{AF} + Y_{FE} + Y_{EK} + Y_{KJ} = 0.$$
$$4{,}000 + 0 + FE.\ \tfrac{4}{5} + 0 - 6{,}000 = 0.$$
$$FE = \frac{5}{4}.\ 2{,}000 = 2{,}500.$$

The member FE is in Compression 2,500 pounds.

Consider the load at joint (4). In this case, half the load will be supported by each abutment; i.e., each reaction will be equal to 3,000 pounds.

The member AB may be shown by the Method of Sections to be in Compression 3,750 pounds and the members DC and FE both in Tension 3,750 pounds.

The results of the preceding discussions are tabulated in Fig. 147.

The Dead and Live Load Stresses having been determined independently of one another, the combined effect of Dead and Live Load may now be examined.

As previously pointed out, the Dead Load is composed of the weight of the truss and any fixed objects that may be on the bridge. This being the case, the stresses determined by considering the Dead Load alone will exist when the bridge is not in use. Furthermore, because the Dead Load is always on the truss, there will always exist the tendency to produce these Dead Load Stresses, although, as will be seen, these stresses may be modified, and in some cases reversed, from Tension to Compression by the Live Load.

A plus sign before a value in Fig. 147 indicates Tension, a negative sign indicating Compression.

Considering first the member AB, it is seen from Fig. 147 that this member is always in Compression. The maximum combined stress due to Dead and Live Load is 15,000 pounds, given when the Live Load is over joint (2) (Fig. 141).

Member	Dead Load Stresses	Live Load at 2	Live Load at 3	Live Load at 4	Maximum Stresses
AB	−8750	−6250	−5000	−3750	−15000
DC	+5250	−1250	+5000	+3750	+10250
FE	+1750	−1250	−2500	+3750	+5500

Fig. 147.

Consider the member DC. This member is in Tension 5,250 pounds, due to the Dead Load. The effect the Live Load taking up a position at joint (2) is to destroy 1,250 pounds of the Dead Load Tension.

The Live Load **alone** at joints (3) and (4) places DC in Tension; that is, in either of these positions the Live Load augments the existing Tension due to Dead Load. It is evident that the maximum stress in DC is 10,250 pounds Tension, given when the Live Load is at joint (3).

Consider the member FE. The Dead Load places this member in Tension 1,750 pounds. The Live Load [illegible] to joint [illegible] is seen to destroy 1,250 pounds of this Tension. Reaching joint (3), the Live Load not only destroys the existing Dead Load Tension, but is seen to place FE in Compression 750 pounds. **However, a Tension member due to its slender proportions cannot take up Compression without "buckling."** This being the case, provision must be made for this 750 pounds Compression, which would otherwise "buckle" FE.

The Counterbrace.

Let Fig. 148 represent the panel in which the member FE lies. In this diagram, the line joining A and B represents the member FE.

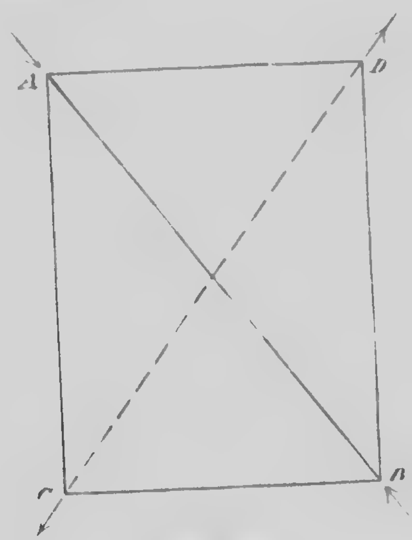


Fig. 148.

It has just been shown that when the Live Load reaches joint (3) (Fig. 141), the member FE would be subjected to 750 pounds Compression. This being the case, the condition of affairs may be represented by a pair of equal forces, one acting at each extremity of FE at A and B as indicated in Fig. 148, these two forces tending to place FE in Compression.

Now, since FE cannot withstand Compression, the joints A and B will tend to move inward under the forces indicated. But when the joints A and B move inward,

the joints D and C will move outward. A Tension member placed from D to C, as indicated by the dotted line, will prevent D and C from moving outward, and, in consequence, prevent A and B from moving inward. That is, a Tension member placed from D to C capable of taking 750 pounds stress will relieve the member FE from its liability to being placed in Compression when the Live Load is at joint (3).

The member FE must be capable of taking 5,500 Tension, which is the maximum combined Dead and Live Load Stress. This stress occurs when the Live Load is at joint (4).

The above reasoning on the counterbrace, while of an almost self-evident nature, must be recognized by the reader as merely a popular explanation, which could hardly stand a scientific examination without further explanation.

The reader can easily see that it would be impractical to place a small compression member alongside FE to take up the excess Compression when the Live Load is at joint (3). If, however, a diagonal member be placed from the remaining corners of the panel, it is evident that **if FE be considered as not acting, and, therefore, left out of the problem,** that this new diagonal member will be in Tension when the Live Load arrives at joint (3). To prove this statement, **let the reader take out the member FE in Figs. 133 and 141** and replace it with a diagonal member placed from the other corners of the panel. If the Dead and Live Load stresses be determined for this new member and then combined, the member will be found in Tension 750 pounds.

Influence Lines.

A curve which shows the variation of stress in any particular member of a truss as the Live Load moves over the structure is known as an Influence Line.

The above definition is complete in as far as it goes, but it may be pointed out here that curves which show the variation in Shear and Bending Moment at any particular point or section of a body for a moving load are also known as Influence Lines.

Curves which show the variation of Stress, Shear or Bending Moment for different sections of a body while the load stays in one particular position are not called influence lines, but have names applied to them according to the function they represent.

APPENDIX.

This portion of the book, presented as an appendix, gives an extremely neat method of solving problems which resuscitate the use of the Funicular Polygon. The reader must not confuse the polygon derived by this method with the frame of the Equilibrium Polygon, although, as far as mere geometrical outline is concerned, both are identical. In the following discussion the original body acted upon is considered as indefinitely extended, and the original forces are replaced by components which by their position counteract one another, thereby allowing the reader to reduce the forces to an equivalent pair. The polygon so derived is, for want of a new name, called by the writer a "Funicular Polygon," although it is fully recognized that, so applied, the name is an absolute misnomer.

THE FUNICULAR POLYGON—SECOND GRAPHICAL CONDITION OF EQUILIBRIUM.

Graphical Location of the Resultant of a Set of Forces.

It has been already shown that the resultant of a set of forces which act at a point also acts at the same point, thus making the problem of locating the resultant a very simple one for that particular case. There arise, however, numerous cases where the forces comprising a set do not act at a point, so that the previous method of locating the resultant does not apply. If the lines of action of the forces comprising the set intersect within reasonable limits, the resultant might be located as follows:—

Let P, Q, S, and T (Fig. 30) represent a set of coplanar forces. Substitute for P and Q their resultant R_1. R_1, S, and T are then equivalent to the original set. Replace R_1 and S by their resultant R_2. R_2 and T are also equivalent to the original set of forces. Substitute for R_2 and T their resultant R_3, which must then be the equivalent of the original forces P, Q, S, and T; i.e., R_3 is the required resultant. Thus by a process of combination and elimination, the resultant of a set of forces may be located. When, however, some or all of

[illegible]

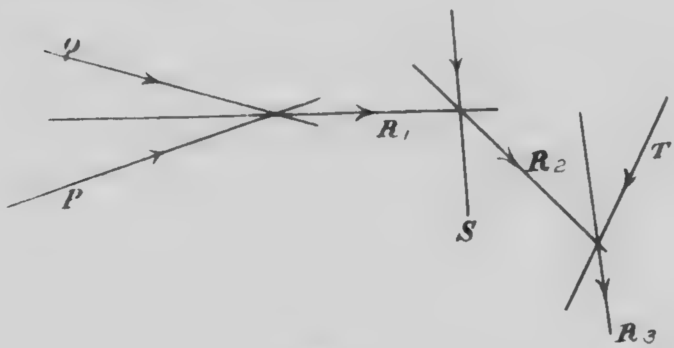


Fig. 30.

Before proceeding with the argument, there are two facts that must be borne in mind throughout this proposition:—

(1) A force may be replaced by its components at any point on its line of action.

(2) The resultant of two forces acts through the intersection of the lines of action of the two forces.

Referring back to Fig. 31, AO and OB represent a pair of components of the force represented by AB; i.e., the force P. P may, therefore, be replaced by AO and OB at any point on its line of action. [See (1).] At any point X on the line of action of P introduce AO and OB. Produce the line of action of OB to intersect the line of action of Q at Y. Q may be replaced by the two components represented by BO and OC, and, since it may be replaced at any point on its line of action, choose the point Y. At Y introduce the components BO and OC. The line of action of BO will, of course,

coincide with the line of action of the component OB of the force P. Produce the line of action of OC to intersect the line of action of S at W, and at W replace S by its two components represented by CO and OD.

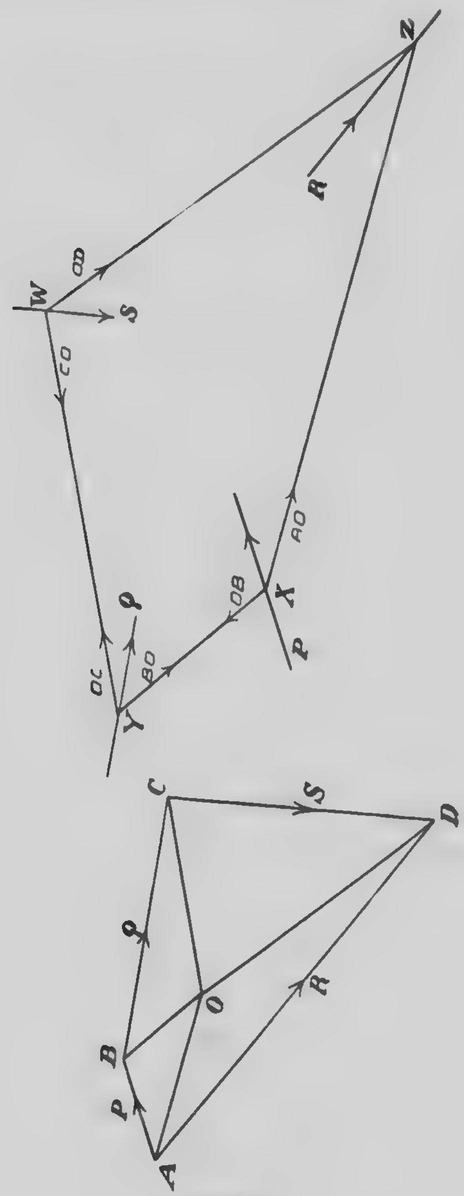


Fig. 31.

The forces P, Q, and S have been replaced by their respective components, so that in place of the original forces there are now acting the forces AO, OB, BO, OC, CO, and OD. It is seen, however, that OB and BO are two forces of equal magnitude, acting with opposite senses in the same line of action. Their effect on the equilibrium of the body is, therefore, zero. In the same way, OC and CO counteract one another, thus leaving AO and OD, so that as far as actual effect is concerned **AO and OD are equivalent to the original forces.** But AO and OD are two components of AD, or conversely, AD is the resultant of the two forces, AO and OD; i.e., from (2), AO and OD may be replaced at the intersection of their lines of action (the point Z) by AD. AD, however, is the resultant of the original set of forces which it was required to locate.

The Funicular Polygon.

In Fig. 31 the polygon XYWZ, formed by the lines representing the directions of the replacing components is given the name of a **Funicular Polygon.** This polygon must not be confused in any way with the vector polygon, nor with the lines drawn from the point O to the vector polygon. This last mentioned point O is known as a "pole," which, together with the vector diagram and the lines to O, form a **Polar Diagram.**

The Resultant Couple.

There arise many cases of sets of forces where, if the vector polygon be drawn, it is found to close, and yet the forces are not in equilibrium. For instance, if in Fig. 32 the vector polygon ACB be drawn for the set of forces P, Q, and S as represented, it is found to close. But it is evident that such a set of forces would produce a rotation of the body: that is, there is not equilibrium, although the resultant force is zero.*

It will thus be seen that cases may arise when a set of forces acts upon a body, such that, although what we term the **resultant force** vanishes, there is yet a motion of rotation about some point in the body; and we speak of the forces as equivalent then to a **resultant couple.**

* Three forces to be in equilibrium must act at a point.

Draw the funicular polygon for P, Q, and S, and it will be found that the first and last components do not intersect one another, if their directions are produced, but that they have parallel lines of action.

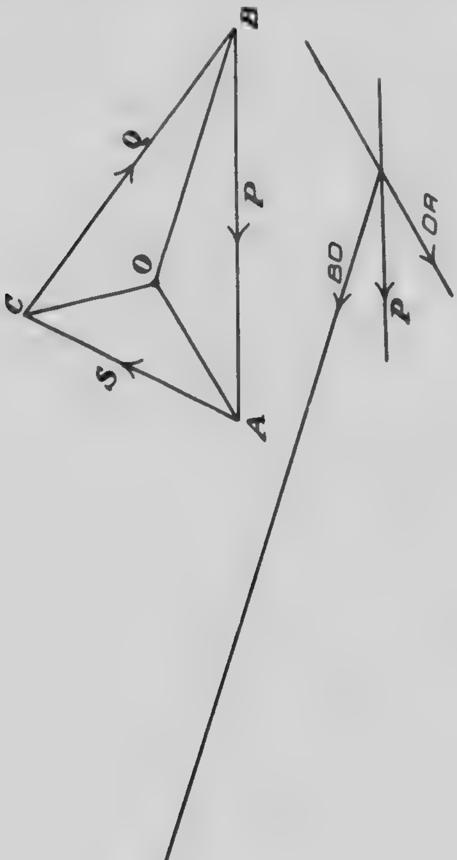


Fig. 32.

Since these first and last components are of equal magnitude and act in the opposite sense, they form a couple, which is called the **Resultant Couple**.

Second Graphical Condition of Equilibrium.

If in the last proposition and in the one preceding it, the lines of action of the first and last components had been coincident, these two components would then counteract one another, so that the effect of all the replacing components would be zero, or, in other words, the original forces would be in equilibrium.

So, then, as a second graphical condition of equilibrium, **the first and last components of the funicular polygon must have their lines of action coincident.**

This is often more concisely stated by saying that **the funicular polygon must close**, keeping in mind what is meant by the word "close."

Graphical and Analytical Determination of the Resultant.

Graphical.	Analytical.
Vector Polygon gives the magnitude, direction, and sense of the resultant.	$\Sigma X = X_R$. $\Sigma Y = Y_R$. $R = \sqrt{\Sigma X^2 + \Sigma Y^2}$. $\tan \alpha = \frac{\Sigma Y}{\Sigma X}$
Funicular Polygon locates the line of action of the resultant.	$\Sigma M = M_R$.

Conditions of Equilibrium.

Graphical.	Analytical.
Vector Polygon must close.	$\Sigma X = 0$. $\Sigma Y = 0$.
Funicular Polygon must close.	$\Sigma M = 0$.

Questions

IN

Applied Statics

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Supplement to APPLIED SCIENCE

Questions in Applied Static

INTRODUCTORY

The following questions have been put together with the idea of helping the student to cover the subject of Applied Statics with the least expenditure of time. In order to bring this about, the outstanding features of the subject have been outlined in these questions, but the reader must clearly understand that a mere answering of the question is not what is alone desired. Question 8, for instance, may be answered by saying that if a set of forces be in equilibrium, the vector polygon must close. This answer is based on the reasoning that the closing line of the vector polygon gives the magnitude, direction, and sense of the resultant, and since a set of forces in equilibrium has no resultant, the vector polygon for this case closes. It is a comprehension of the principle behind the question that is ~~arrived~~ at aimed

The student is urged to remember that in every problem where a set of forces is considered, there must be some body acted upon. Very often this body is not represented, as in the case where a set of forces is represented arbitrarily on a sheet of paper, but nevertheless, there would have to be a body acted upon even in these cases. In such problems as the last mentioned, the representation of the body is left ~~and~~ simply because its shape has no direct bearing on the problem.

In every case, it will save the student a great deal of time if the body acted upon in the problem be kept definitely in mind. If this is done, it can easily be seen, that the body being clearly outlined, the forces acting on it can readily be picked out and the desired information obtained.

It must be recognized that the Method of Sections is not a method particularly to be applied to certain problems, but is, in reality, used in all statical problems. Whenever one considers a set of forces acting on a body, a section has been taken around some portion of material which may even be part of a larger body, and this isolation of a body acted on constitutes the method of sections.

In general, it will be found that the questions follow the order of the lecture course; and to help still further, the numbers of the pages in the text-book, bearing on the principles involved, have been given after each question where possible.

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APPLIED STATICS

If you have not done so, read over the introduction to these questions.

1. What is meant by a Statical Diagram? (9-10)
2. Show how to use Bow's Notation. (10)
3. What is meant by a Resultant Force? (10)
4. Define the Equilibrium or Balancing Force of a set of forces. (11)
5. Is there any relation between the Resultant and the Balancing Force of the same set of forces? (11)
6. Show how to find graphically the magnitude, direction, and sense of the Resultant of any set of forces. (11-13)
7. If a set of forces is in equilibrium, what is the Magnitude of their Resultant? ~~(13)~~ 11+ 15
8. State, then, one graphical condition of equilibrium. (14)
9. What is meant by a pair of Resolved Parts of a force? (14)
10. If the angle of inclination of a force be taken with reference to the horizontal, and if the horizontal and vertical resolved parts of a force be designated by X and Y respectively, then X of force = force x cos angle of inclination, Y of force = force x sine angle of inclination. (17)

The above relations are extremely important. Remember them.

11. In what sense do positive X's and Y's act? The opposite sense is taken as negative. (17)
12. **Show that $\Sigma X = X_R$, $\Sigma Y = Y_R$. (23)**
13. **What is the value of both ΣX and ΣY when the set of forces is in equilibrium? (27)**
14. Define the moment of a force about a point. (22)
15. What convention have you been using as to positive and negative moments? (22)
16. Prove $\Sigma M = M_R$. (24) εm
17. What is the value of ΣM when the set of forces is in equilibrium? (27)
18. State the three analytical conditions of equilibrium. (27)
19. Construct diagrams showing the form of the following trusses: French, Fink, Warren, and Pratt.
20. What is meant by a pin jointed truss? (111)
21. Explain what is meant by the statement: "Consider the forces at a certain joint of a truss." What is the body acted upon? (111)
22. Show how to determine abutment reactions analytically. Take a case of a Warren truss. (32)

23. Having solved the abutment reactions in question 22, determine the stress in a few of the members of the truss by considering the forces acting at the various joints. Begin at the abutment.

24. Show how to apply the method of sections so as to find the stress in any desired member of the same truss without having to work from joint to joint till the desired member is reached. (49)

25. Construct the stress diagram for a Pratt truss supporting 1,000 pounds at each joint. Draw a stress diagram for a Warren truss with a load of 1,000 pounds at each joint. (150)

26. What precaution must be observed when the equations $\Sigma X = o$ and $\Sigma Y = o$ have to be used simultaneously? (109)

27. The Equilibrium or Funicular Polygon represents a pin jointed frame structure which replaces the original body, and its shape gives the desired information. (41)

28. If a set of forces be in equilibrium, what form does the Funicular Polygon take? (45)

29. State the two graphical conditions of equilibrium. (45)

30. Show how to find abutment reactions graphically. (68)

31. Define the Vertical Shearing Force and the Bending Moment of any section in a beam. (55 and 57)

These two definitions must be remembered and understood.

32. A beam 40 feet long supports three concentrated loads of 400 pounds placed at 10, 20, and 30 feet from the left abutment. Find the V. S. F. and B. M. at sections 5, 10, 16 and 31 feet from the left abutment.

33. A beam 20 feet long supports a uniformly distributed load of 200 pounds over the right-hand half and a concentrated load of 100 pounds at a distance of 5 feet from the left abutment. Find the V. S. F. and B. M. at sections 5, 9, 11, and 15 feet from the left abutment.

34. Construct the V. S. F. diagrams for questions 32 and 33.

35. What is meant by the statement that the ordinate of the Funicular Polygon represents the B. M.?

36. What is the mechanical advantage of a system of pulleys? (78)

37. Show how to arrange a double and a single block so as to most advantageously lift a given weight. (79-80)

38. Deduce an expression for the mechanical advantage of a Weston Differential pulley. (82-84)

39. What is meant by the Angle of Repose? (89)

40. State the relation between the Angle of Repose and the Co-efficient of Friction. (89)